

Sense Making: Is it at the Core of Your Classrooms?

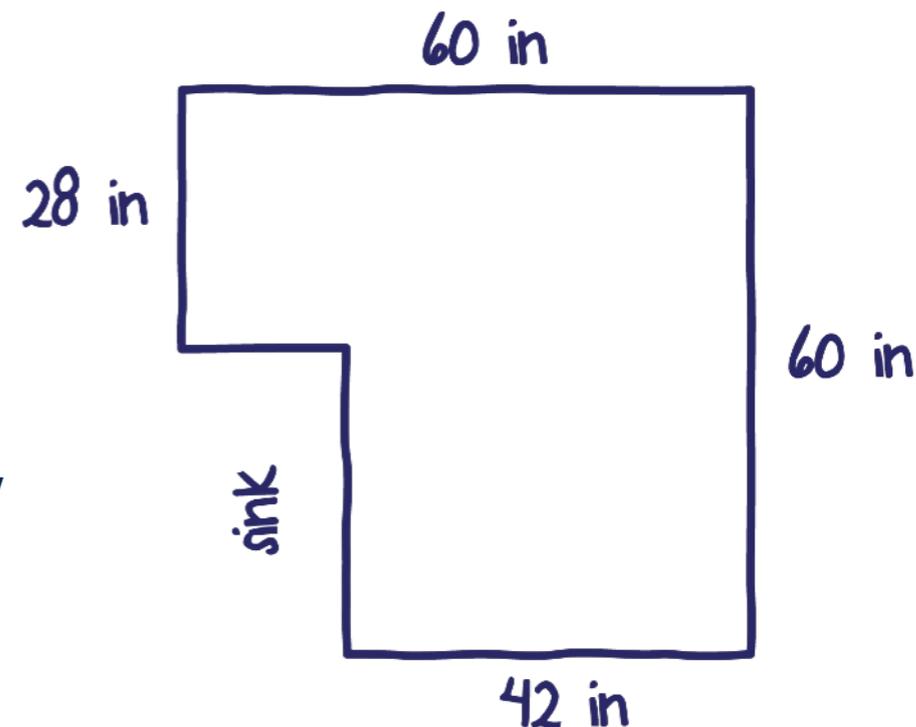
Annie Fetter, @MFAnnie, anniefetter@gmail.com

CAMT 2018, Houston

**A PDF of the slides will be available after the session at
annie.mathematicalthinking.org**

Teresa's Tiles

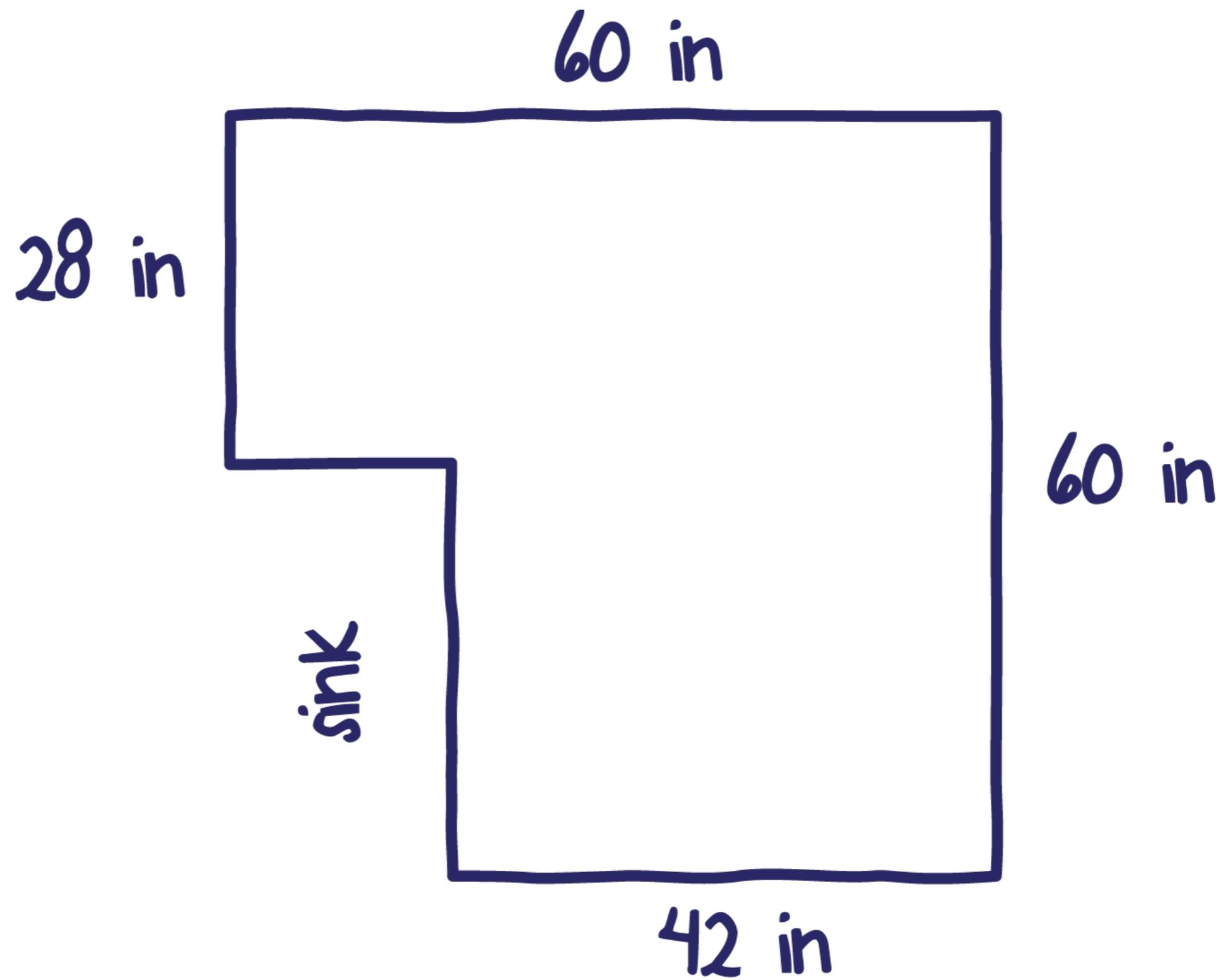
Teresa is going to put down new ceramic tiles on her bathroom floor. She has selected square tiles that are 4 inches on each side. These are the kind of tiles that can be placed right next to each other without leaving additional space for grout. At The Home Station, she learned how to cut the tiles in case she needs any fractional pieces to cover her floor completely.



This diagram of the bathroom floor shows the dimensions of the floor space she needs to cover. The sink area does not get tiled.

Questions: How many tiles will she need to buy to cover her floor? How many tiles will she have to cut in order to cover the entire space?

Extra: What is the size, using whole numbers, of the largest square tile that could be used to tile the entire floor with no cut pieces?



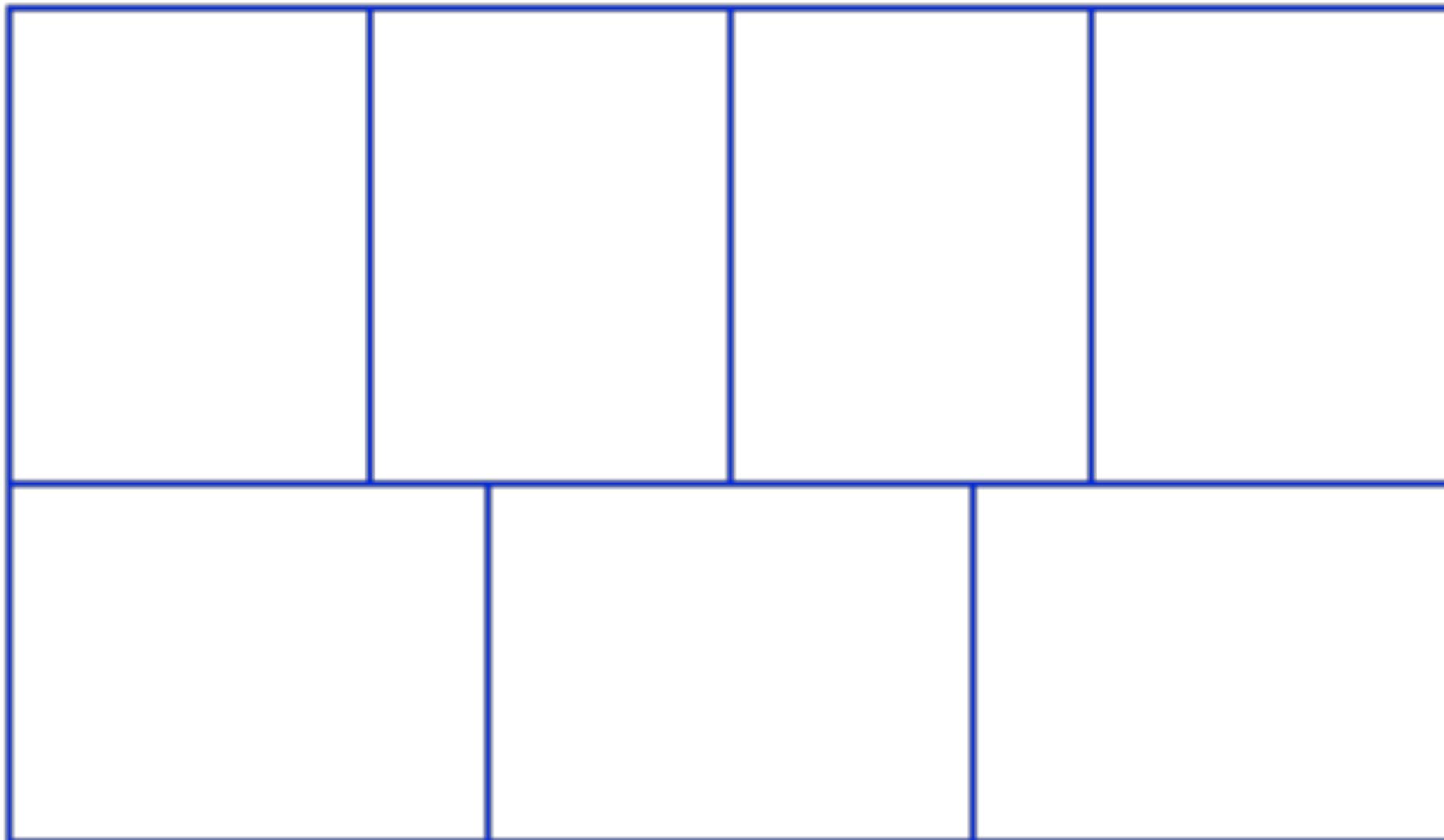
Teresa's Tiles "Scenario"

Things that some "low-performing" 8th graders noticed about the picture:

- two sides are equal
- two sides are 60 inches
- one side is 28 inches
- they are longest
- one side is 42 inches
- it used to be a square
- your lines aren't very straight
- the short side of the sink is 18"
- the sink is a rectangle
- the long side of the sink is 32"
- can find the area of the whole thing by making it two pieces

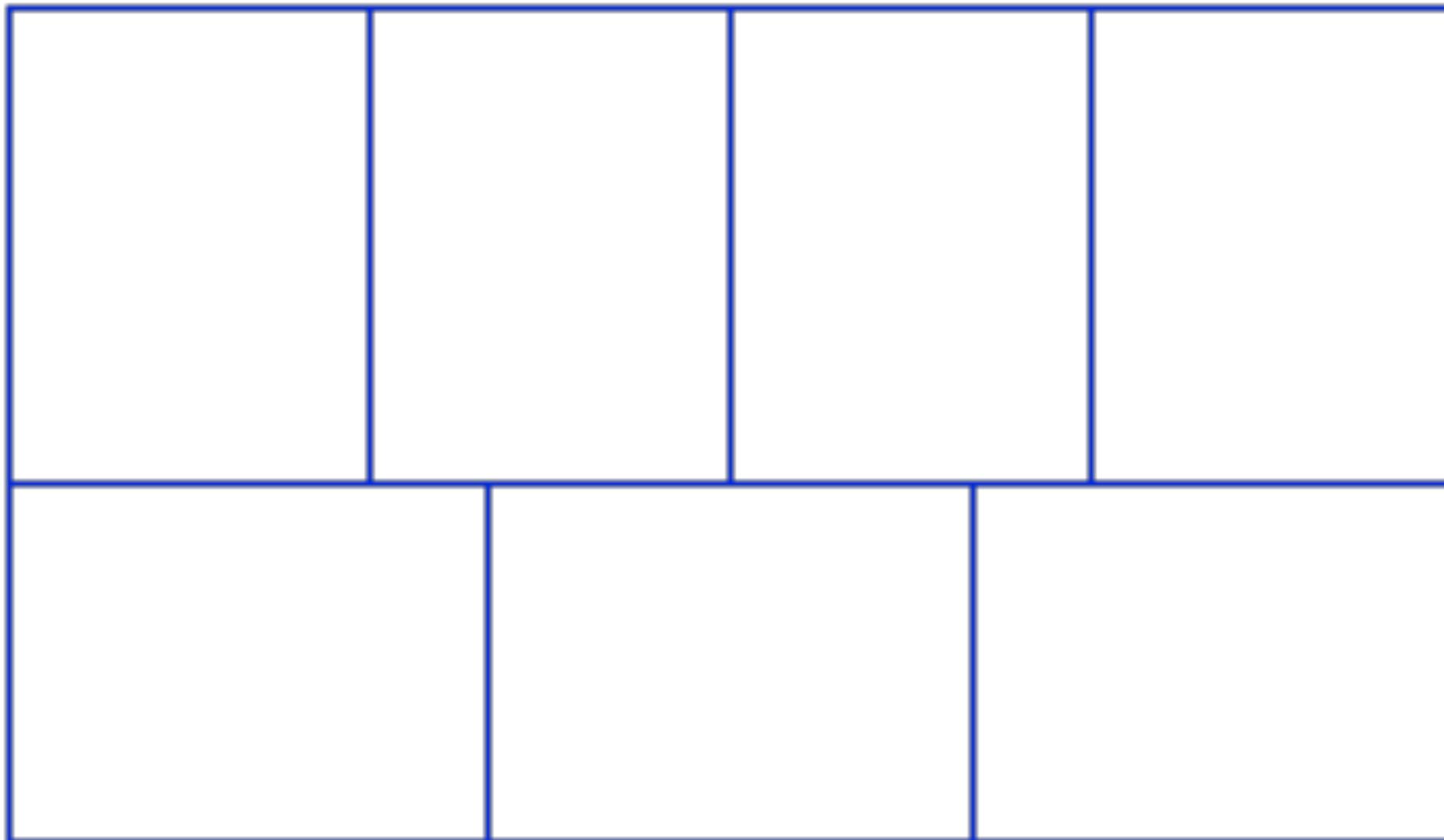
Congruent Rectangles “Scenario”

The seven small rectangles in this picture are congruent.



Congruent Rectangles “Scenario”

The seven small rectangles in this picture are congruent.
The area of the large rectangle is 756 square centimeters.



**There are 125 sheep and
5 dogs in a flock.
How old is the shepherd?**

Robert Kaplinsky (@robertkaplinsky) gave this to 32 eighth grade students. What percentage do you think tried to give numerical answers?

How Old is the Shepherd?

75% gave numerical answers.

100% of his sixth graders gave numerical answers.

In the original research paper [Reusser 1986],
“...three out of four school children will produce a numerical answer to this problem.”

<http://robertkaplinsky.com/how-old-is-the-shepherd/>
<https://www.youtube.com/watch?v=kibaFBgaPx4>

Sample Grade 3 Test Question

The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?

- A. 6 18%
- B. 9 46%
- C. 18 31%
- D. 24 4%

Combined scores of the 160 third graders in a group of four low-performing schools I used to support.

Sample Test Question Revised

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.



TEKS Mathematics

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

TEKS Mathematics

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

Some Random Math Habit of Mind

Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt.**

They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

They monitor and evaluate their progress and change course if necessary.

Sample Grade 3 Test Question

Hot dog buns come in packages of 8. Michael buys 6 packages of hot dog buns. How many hot dog buns does Michael have in all?

- A. 14 43%
- B. 36 8%
- C. 48 40%
- D. 56 5%

“Cracking the Math Code”

ADDITION

Add
Altogether
And
Both
How many
How much
In all
Increased by
Plus
Sum
Together
Total

SUBTRACTION

are not
change
decreased by
difference
fewer
have left
how many did not
have
how many more
less than
remain
subtract
take away
Taller/shorter

MULTIPLICATION

By (dimension)
Double
Each group
Multiplied by
Of
Product of
Times
Triple

DIVISION

as much
cut up
divided by
each group has
half (or other
fractions)
how many in each
parts
quotient of
Separated
Share something
equally
split

(document from the web site of a large eastern metropolitan school district)

Sample Grade 3 Test Question

Hot dog buns come in packages of 8. Michael buys 6 packages of hot dog buns. How many hot dog buns does Michael have in all?

- A. 14 43%
- B. 36 8%
- C. 48 40%
- D. 56 5%

Integrated Math 2 (Grade 10)

	Boy	Girl	Total
Wear Sunscreen	84	133	217
Do Not Wear Sunscreen	170	118	288
Total	254	251	505

$P(\text{wears sunscreen})$

$P(\text{is a boy})$

$P(\text{wears sunscreen} \mid \text{is a boy})$

$P(\text{wears sunscreen and is a boy})$

$P(\text{wears sunscreen or is a boy})$

$P(\text{is a boy} \mid \text{wears sunscreen})$

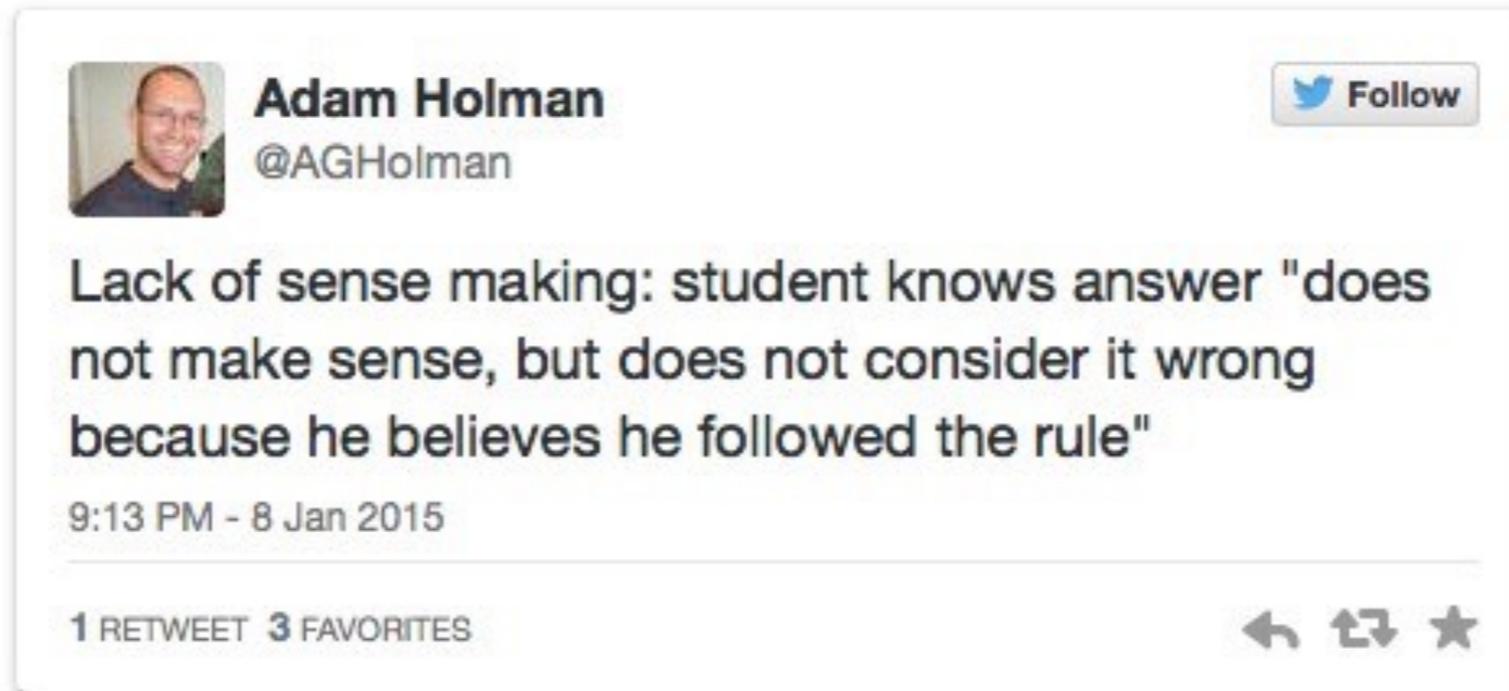
$P(\text{is a boy or a girl})$

“Doing Math” or Sense Making?

$$12 - p = 5$$

$$12 - ? = 5$$

"The Steps" *Override* Thinking



[Michelle's son] was struggling to “remember” $28/4$. When [she] asked him, “How do you think about $28/4$?” He replied, “Mom, you aren’t supposed to think about it, you are just supposed to do it!!”

Teacher Knows Best

$$\frac{3}{4} + \frac{5}{8} = \frac{8}{12}$$

Student Perceptions of Math and Sense Making

1. You aren't supposed to sense-make when doing math.
2. You are supposed to use rules and algorithms and accept whatever answer results.
3. You are supposed to do what your teacher said, even when it doesn't seem like a good idea.

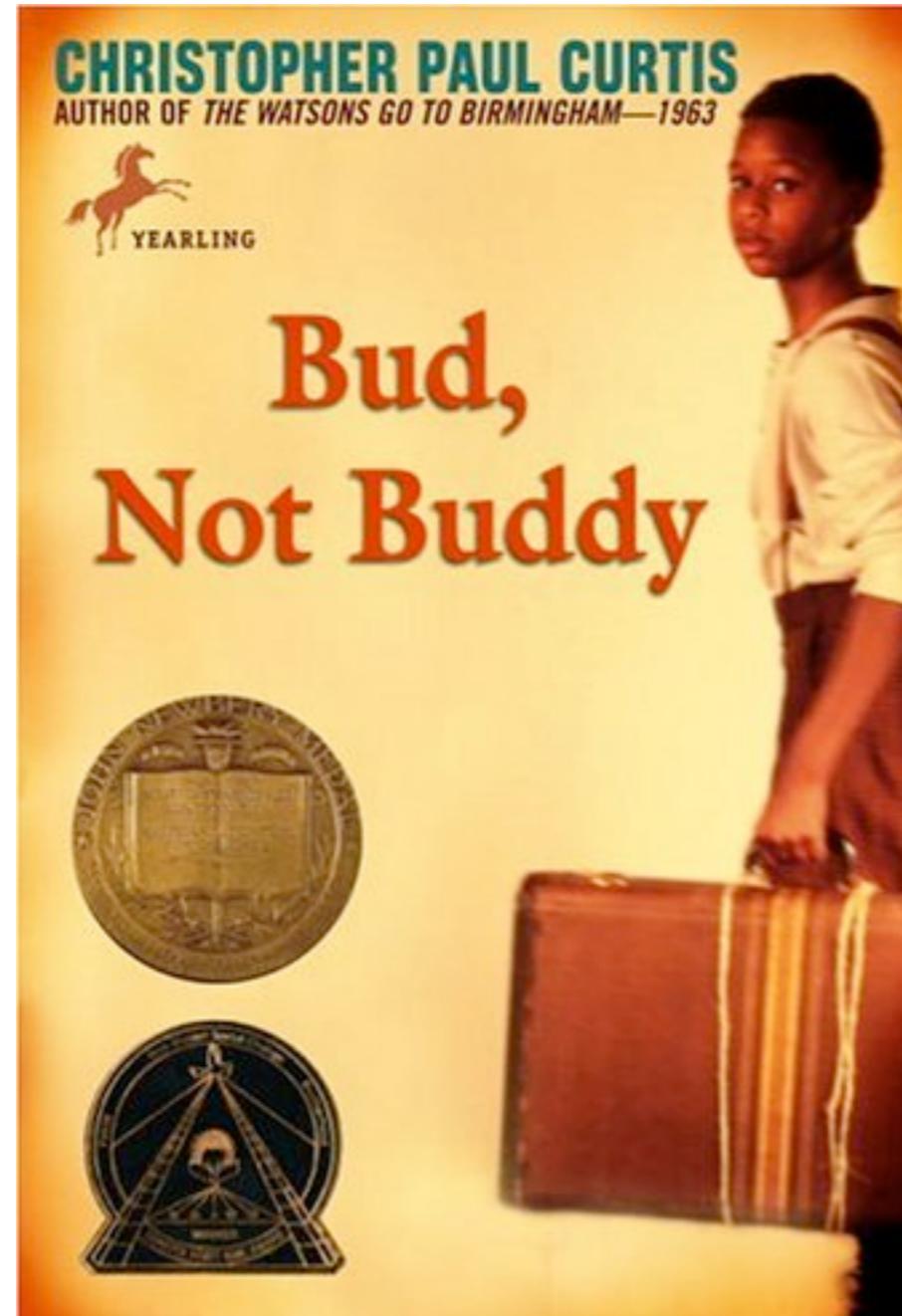
The Five Strands of Mathematical Proficiency

National Research Council, 2001, *Adding it up: Helping children learn mathematics*.

1. Conceptual understanding
2. Procedural fluency
3. Strategic competence
4. Adaptive reasoning
5. Productive disposition

“Productive disposition is the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.”

Jekyll and Hyde?



Characteristics of Strong ~~Readers~~ *Mathematicians*

- They are motivated to read. ~~read~~ *tackle problems*
- They are able to ~~read words~~ accurately and automatically. *recite facts*
- They comprehend what they read.
- They are able to read with expression.
- They use a variety of strategies to tackle ~~words~~ they don't recognize. *problems*
- They use active problem solving strategies to search for information, to determine meaning, to make sense of words, to make connections.

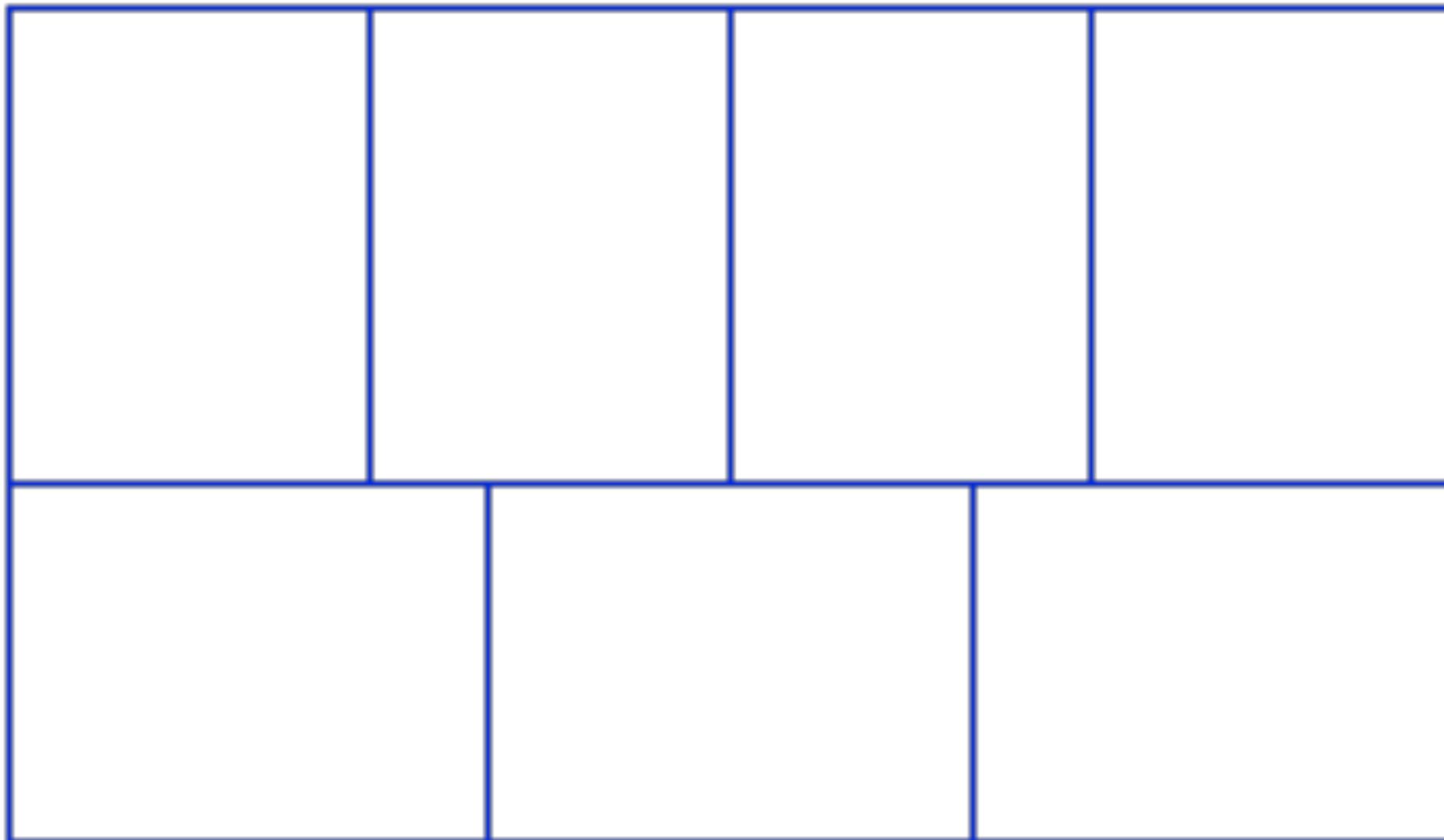
Encouraging Sense Making

Q: What's one way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question. Literally.

Congruent Rectangles “Scenario”

The seven small rectangles in this picture are congruent.
The area of the large rectangle is 756 square centimeters.



Get Rid of the Question

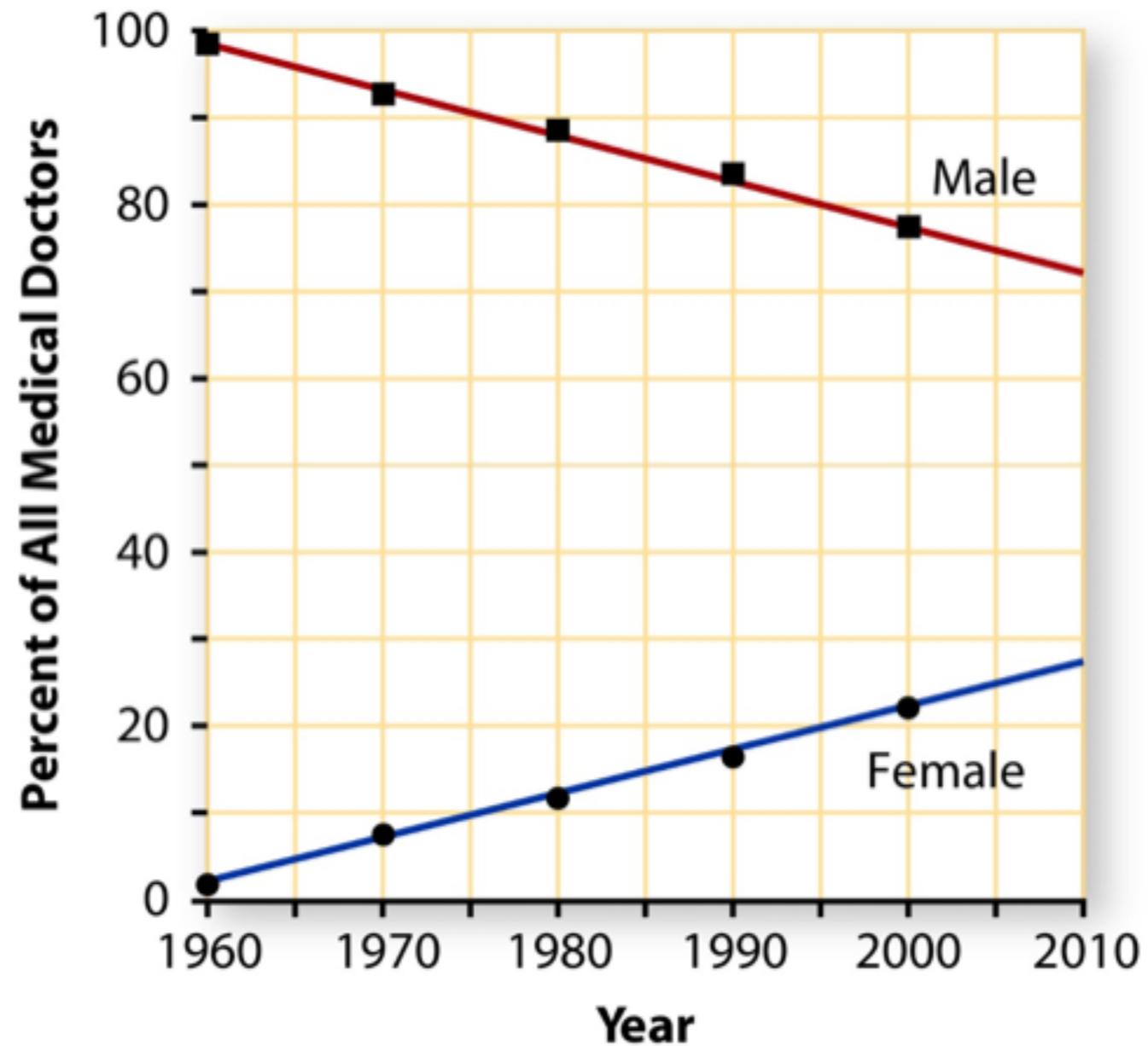
Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

I Notice

I Wonder

Get Rid of the Question

Male and Female Medical Doctors

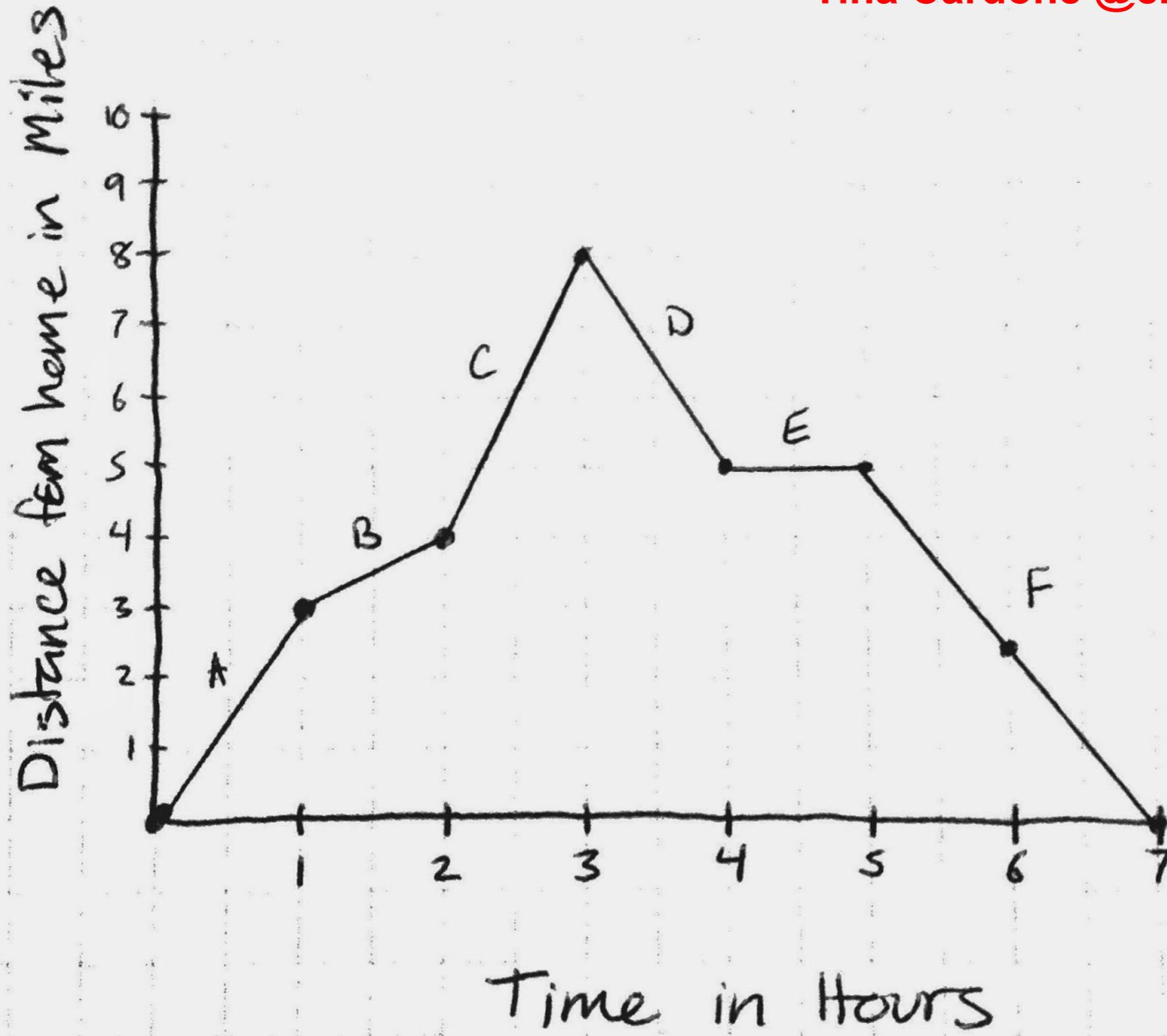


Get Rid of the Question

Think About This Situation

Study the trends in the percentage of male and female medical doctors in the United States between 1960 and 2000.

- a** How would you describe the trends shown in the data plots and the linear models that have been drawn to match patterns in those points?
- b** Why do you suppose the percentage of women doctors has been increasing over the past 40 years?
- c** Would you expect the trend in the graph to continue 10 or 20 years beyond 2000?
- d** How would you go about finding function rules to model the data trends?
- e** If you were asked to make a report on future prospects for the percentages of male and female doctors, what kinds of questions could you answer using the linear models?





Tina Cardone @crstn85 · Nov 24

@MFAnnie when I gave the graph and did notice/wonder first I didn't have to answer nearly so many questions when they did the handout



Tina Cardone @crstn85 · Nov 24

@MFAnnie worth the few minutes it took and meant we skipped wrap up discussion (they already had it)
drawingonmath.blogspot.com/2014/11/distan...



<http://drawingonmath.blogspot.com/2014/11/distance-graph.html>

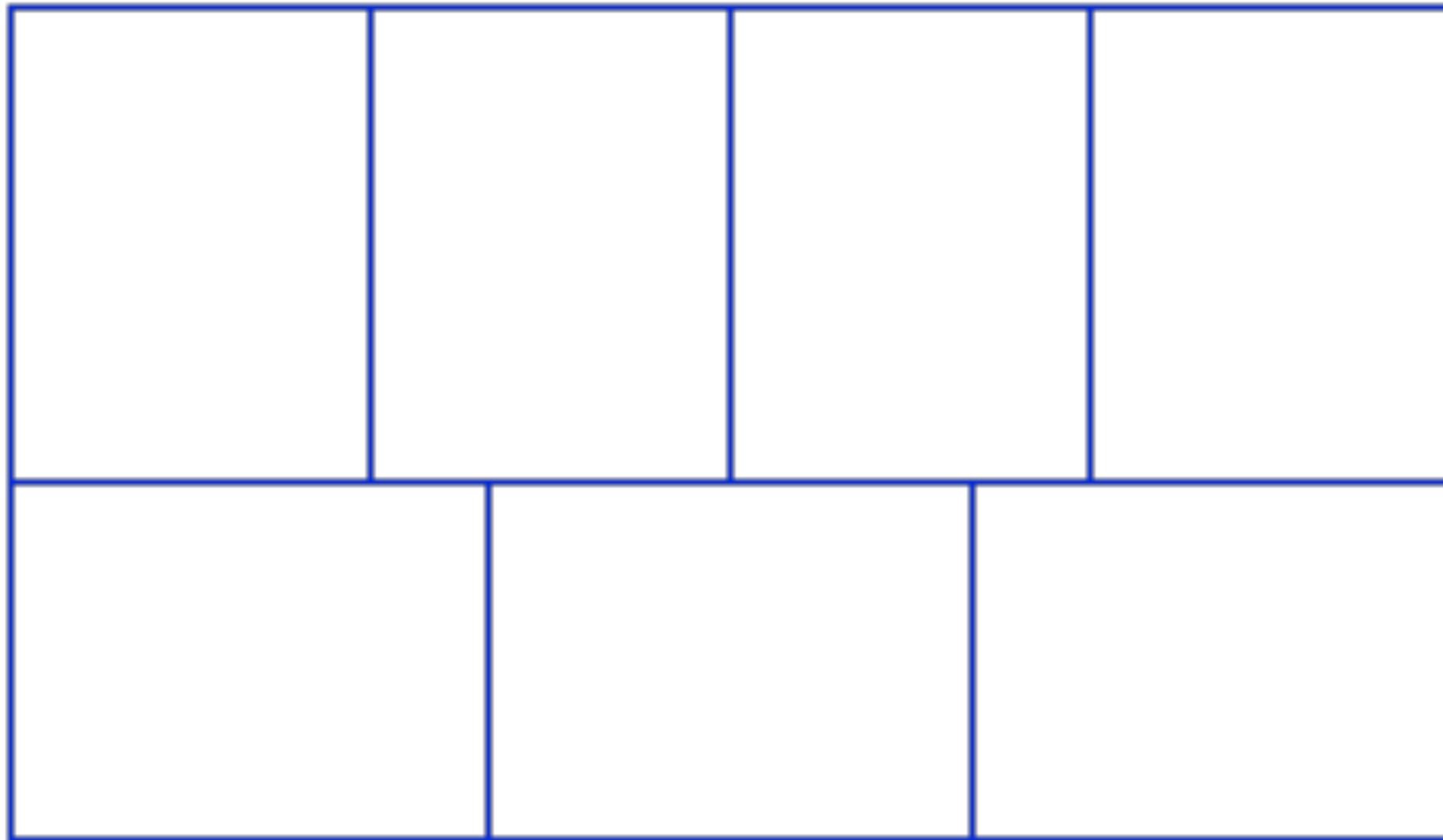
Encouraging Sense-Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question *and* the numbers.

Congruent Rectangles “Scenario”

The seven small rectangles in this picture are congruent.

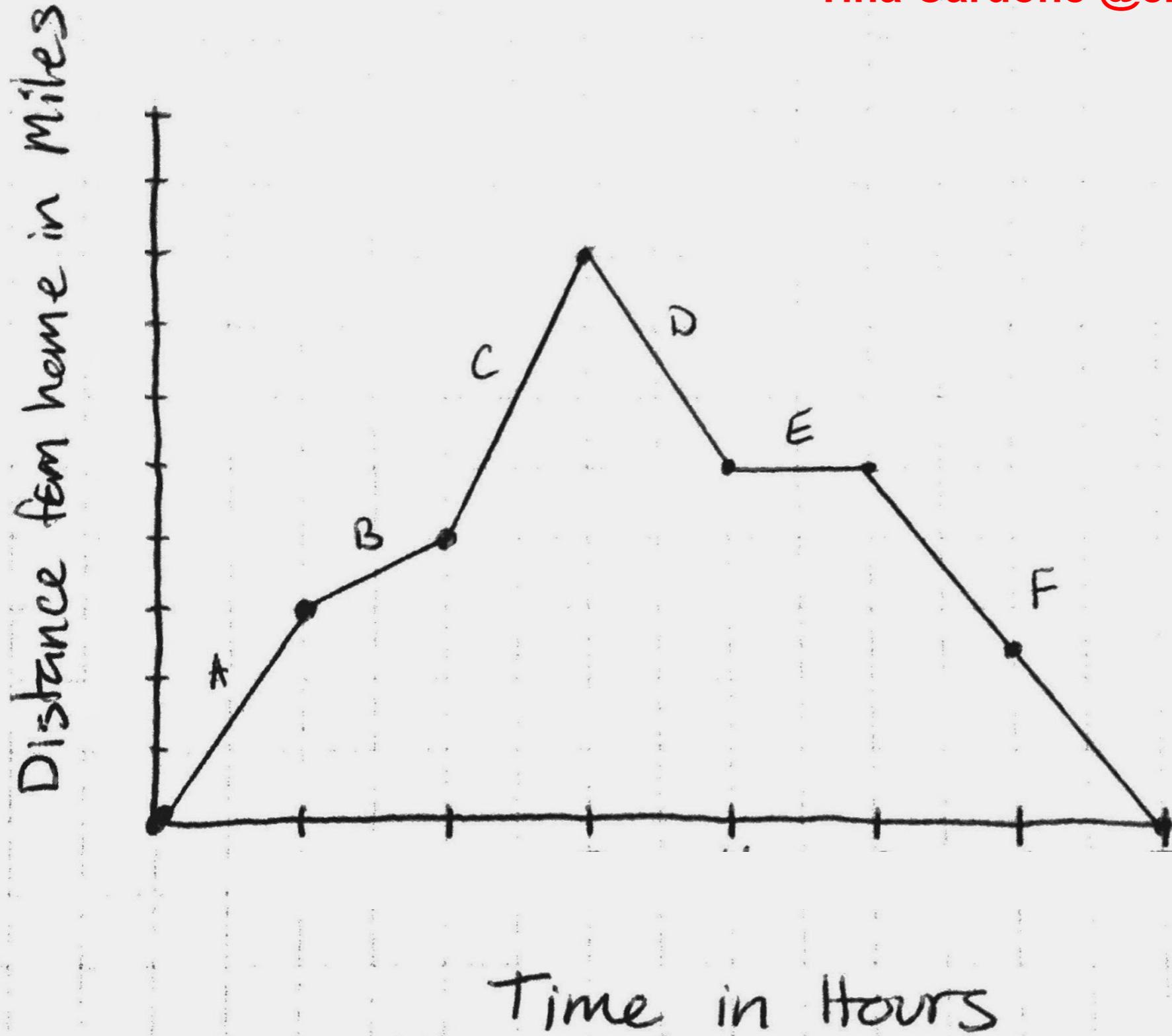


Get Rid of the Question and the Numbers

Raul had some pet mice. Xavier gave him some more mice.
Raul had some pet mice. Xavier gave him 3 more mice.
Raul had some pet mice. Xavier gave him 3 more mice.
Now Raul has 8 mice.

Raul had some pet mice. Xavier gave him 3 more mice.
Now Raul has 8 mice. How many mice did Raul have to
start with?

A Numberless Word Problem from Brian Bushart, bstockus.wordpress.com



Get Rid of the Question and the Numbers

A store has the floor plan shown. The area of the women's department is



Numbers vs. Relationships

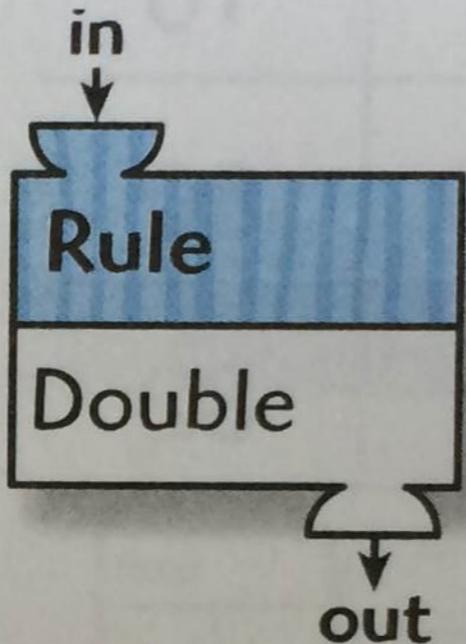
$$A = \pi r^2$$

Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Give the answer.

Give the Answer



in	out
8	16
50	100
200	400
75	150
150	300

Give the Answer

◆ Math Message Follow-Up

WHOLE-CLASS ACTIVITY

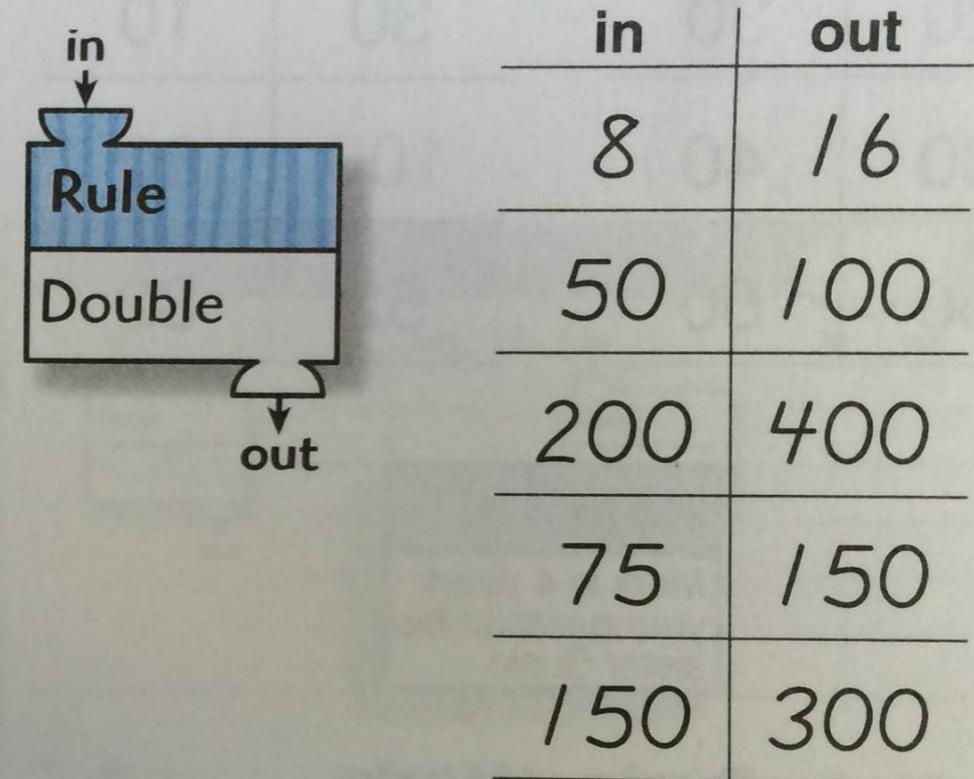
Draw or display a function machine and “What’s My Rule?” table. (See Advance Preparation.)

Ask children to imagine that the **function machine** works like this:

- A number (the **input**) is dropped into the machine,
- the machine changes the number according to a rule,
- and a new number (the **output**) comes out the other end.

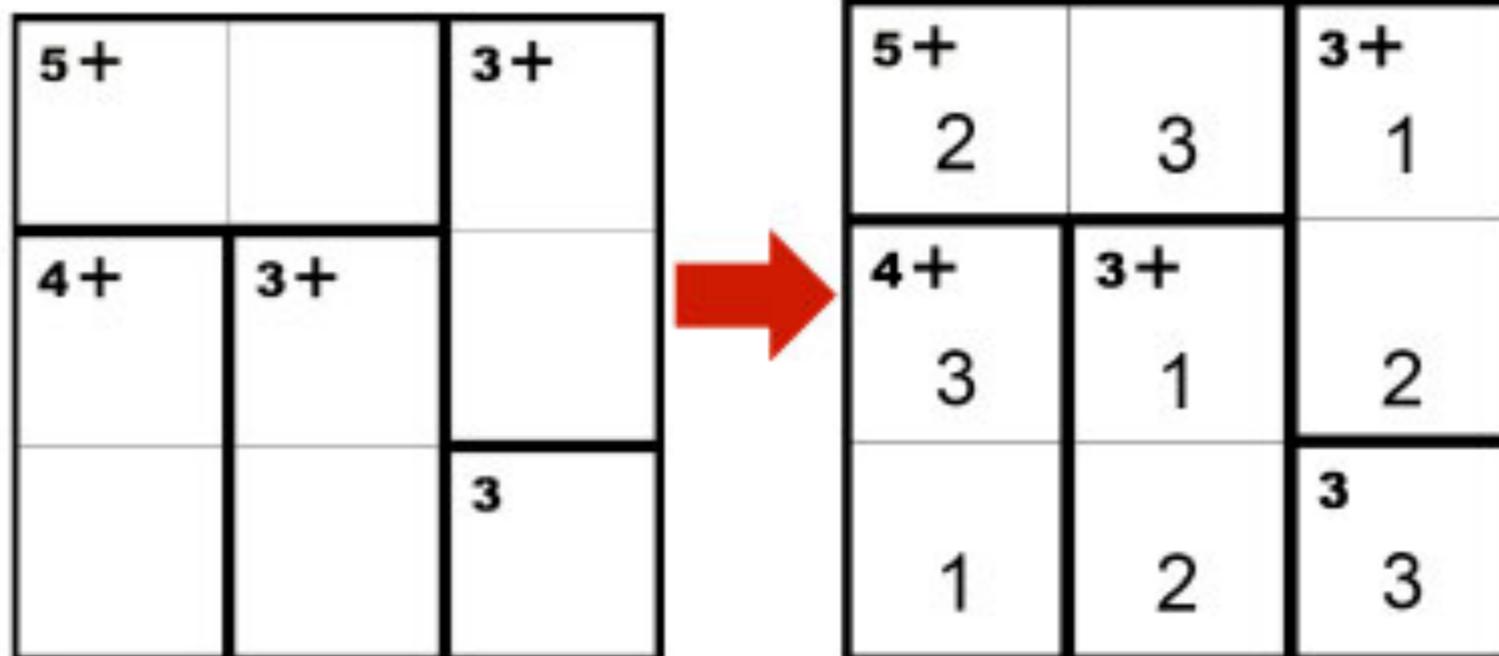
The **rule** for the Math Message problem is “Double the number.” Write the word *Double* in the function machine.

Point out the “**What’s My Rule?**” table. Discuss the 8 in the *in* column and the 16 in the *out* column. Explain to children that numbers in the *in* column represent the numbers of bacteria now. Corresponding numbers in the *out* column represent the numbers of bacteria 20 minutes from now.



How to Play KenKen®

Your goal is to fill in the whole grid with numbers, making sure no number is repeated in any row or column.

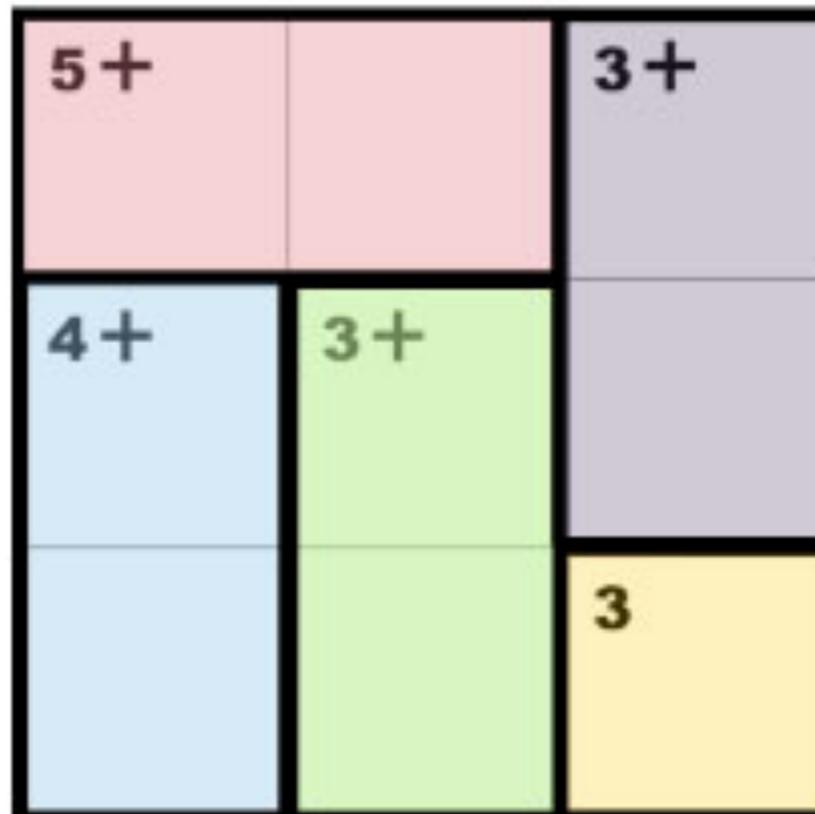


In a 3x3 puzzle, use the numbers 1 – 3.

	1	2	3
1	5+		3+
2	4+	3+	
3			3

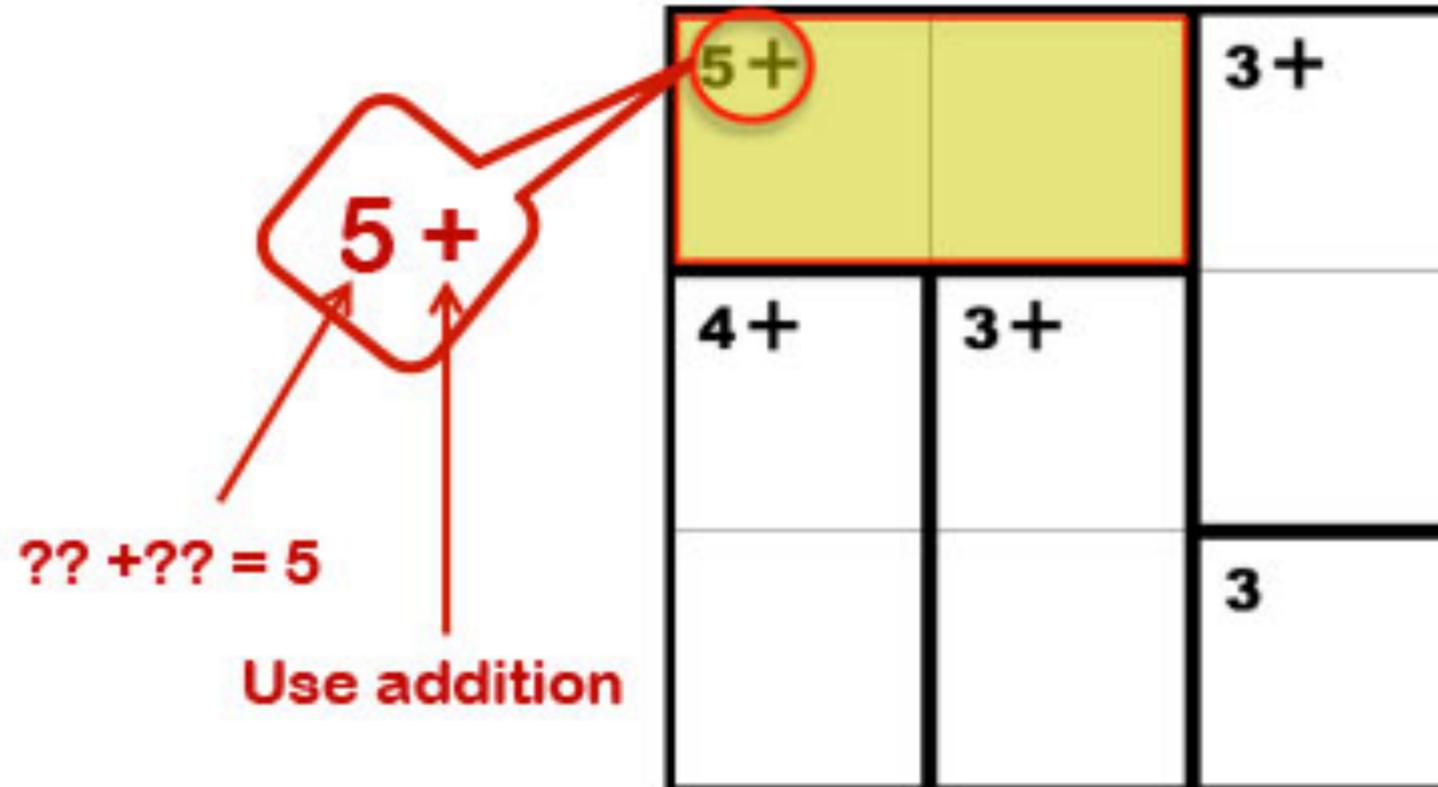
In a 4x4 puzzle, use the numbers 1 – 4.
In a 5x5, use the numbers 1 – 5, and so on.

The heavily-outlined areas are called
“cages.”



This puzzle has 5 cages.

In this cage, the math operation to use is **addition**, and the numbers must add up to **5**. Since this cage has 2 squares, the only possibilities are 2 and 3, in either order (2+3 or 3+2 = 5).



A cage with one square is a “Freebie”...
just fill in the number you’re given.

5+		3+
4+	3+	
		³ 3

A number cannot be repeated within the same row or column.

X

5+		3+
4+	3+	
3		³ 3

$5+$ 2	3	$3+$ 1
$4+$ 3	$3+$ 1	2
1	2	3 3

Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Ask about ideas, not answers.

Ask About Ideas, Not Answers

This can be really simple:

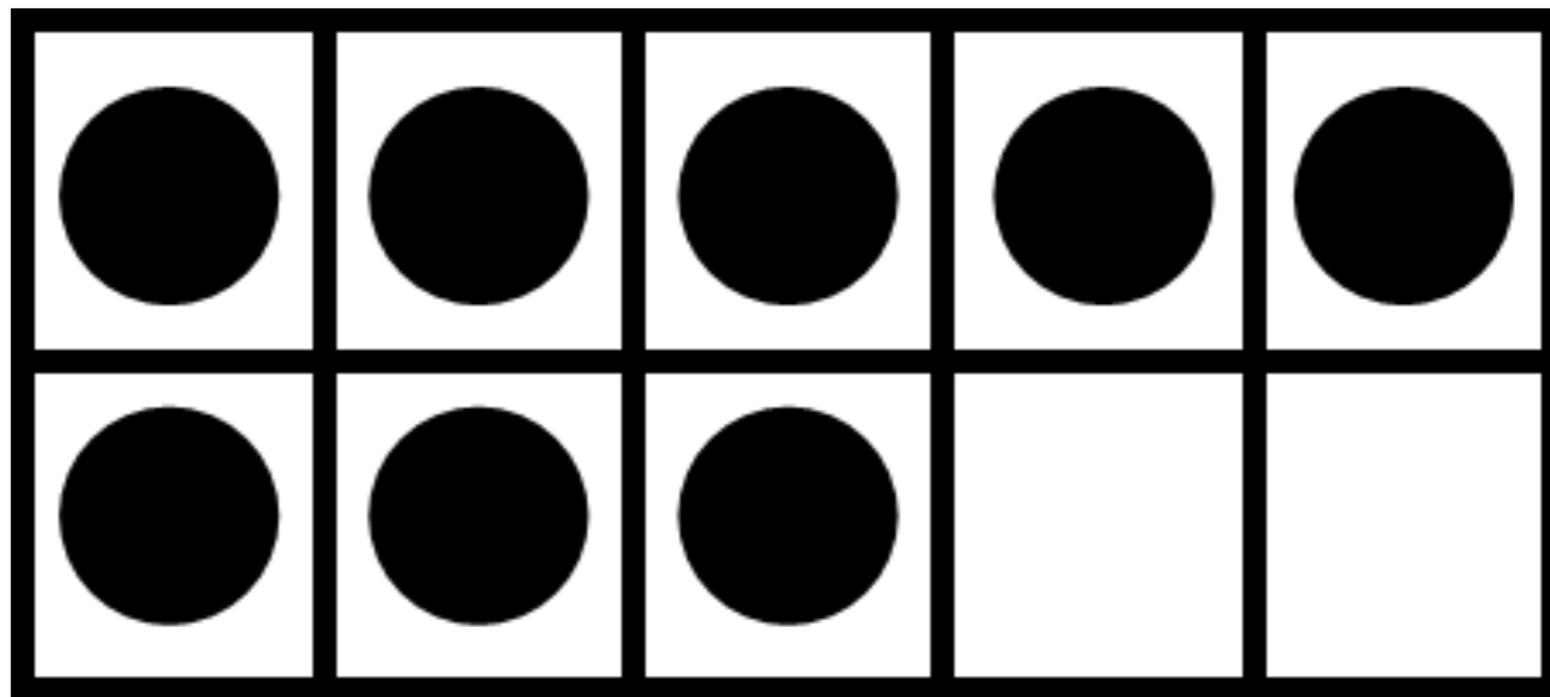
“Tell me something about number 7.”

instead of

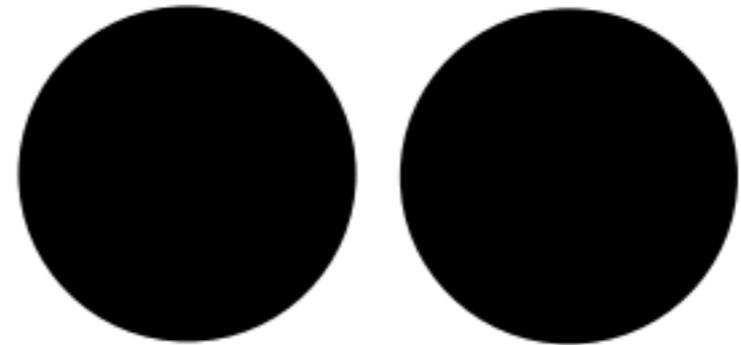
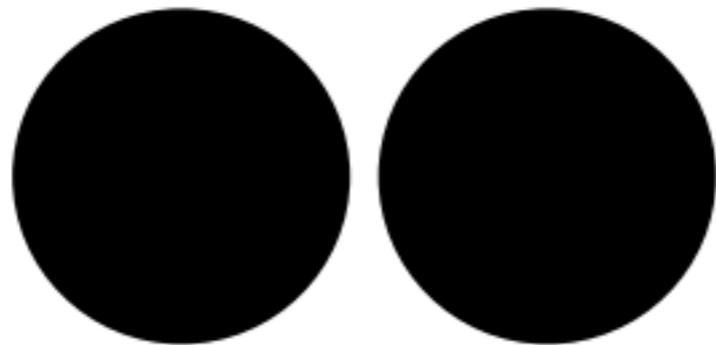
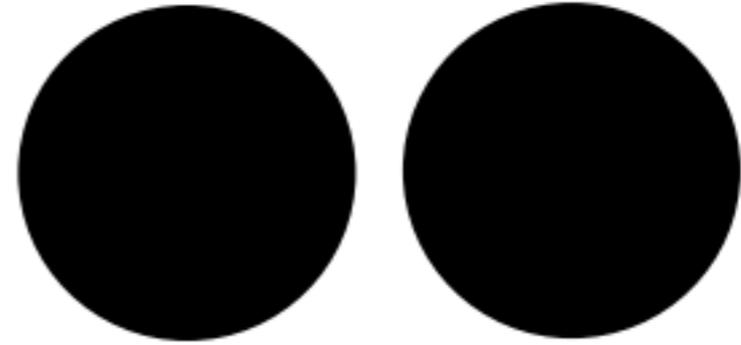
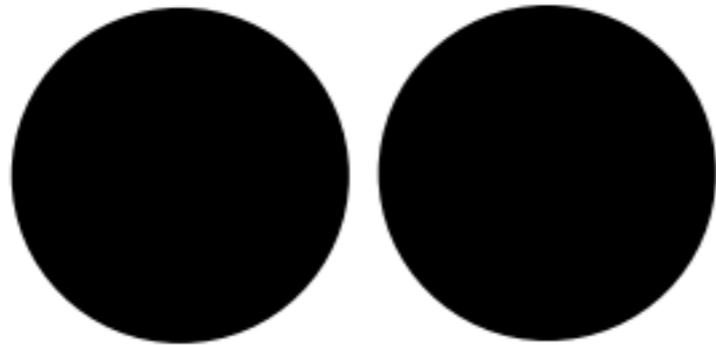
“What’s the answer to number 7?”

Ask About Ideas, Not Answers

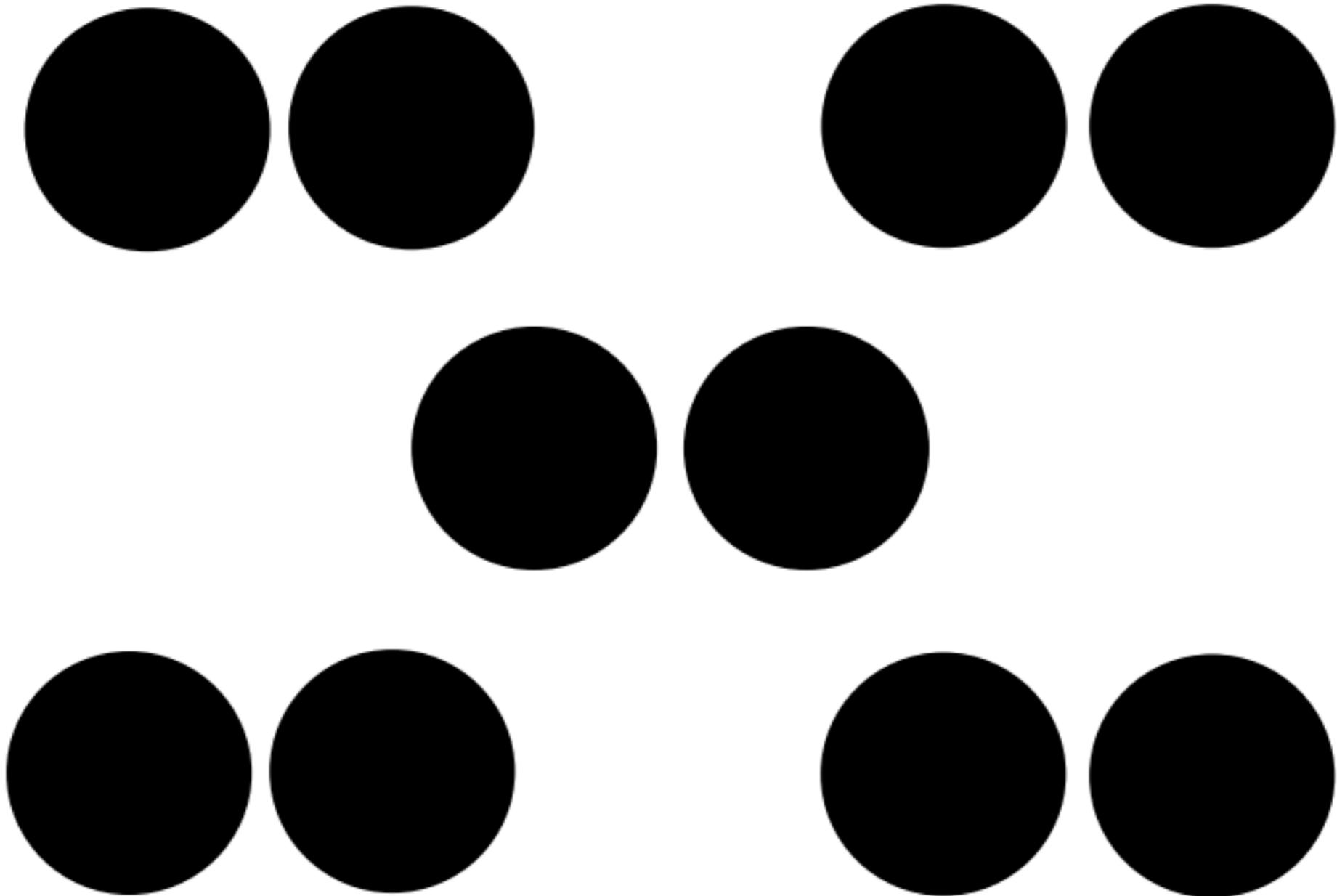
It can be all about the ideas, not the answer.



Ask About Ideas, Not Answers



Ask About Ideas, Not Answers



Ask About Ideas, Not Answers

It can be all about the ideas, not the answer.

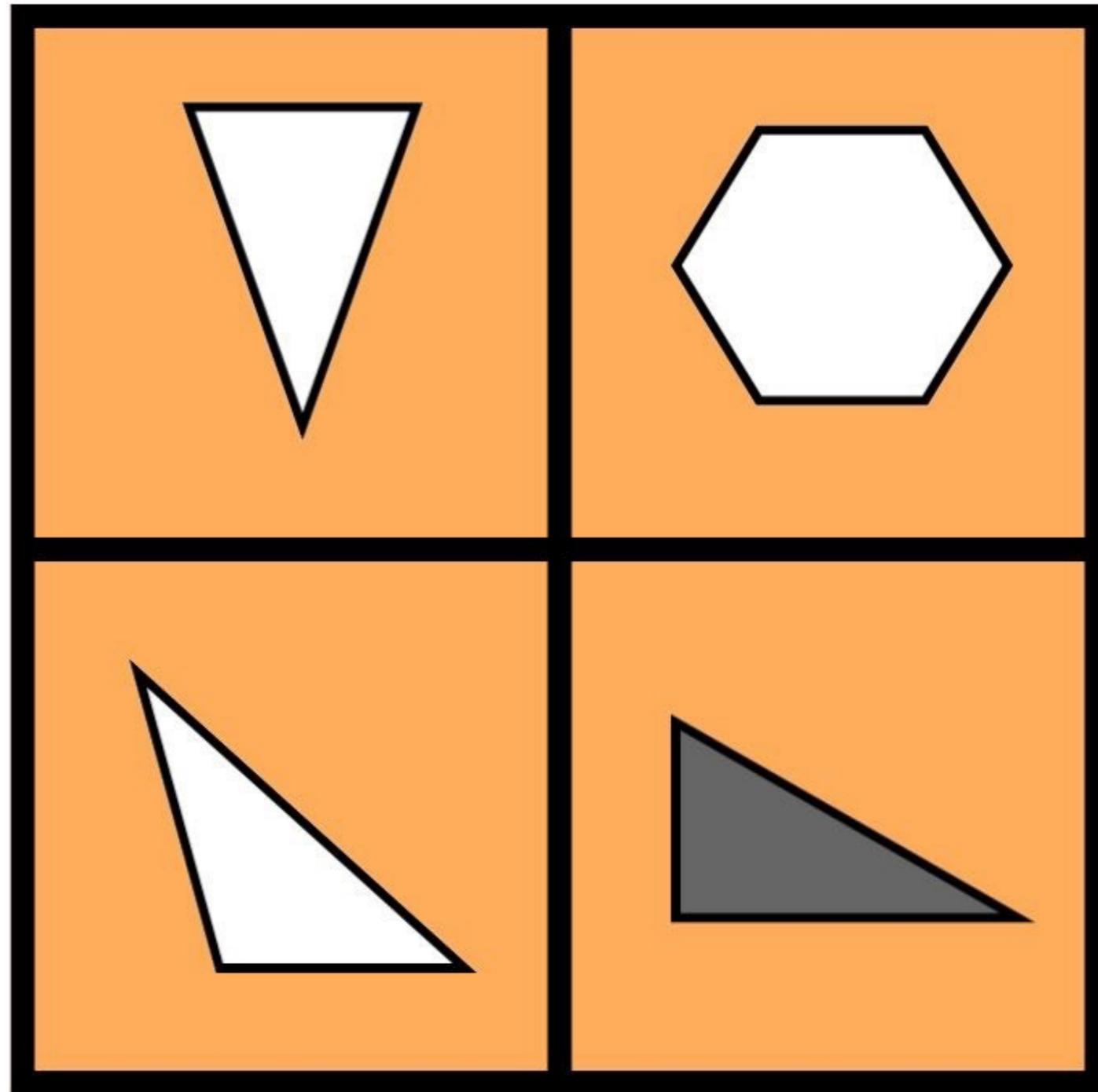
$$119 + 36$$

Ask About Ideas, Not Answers

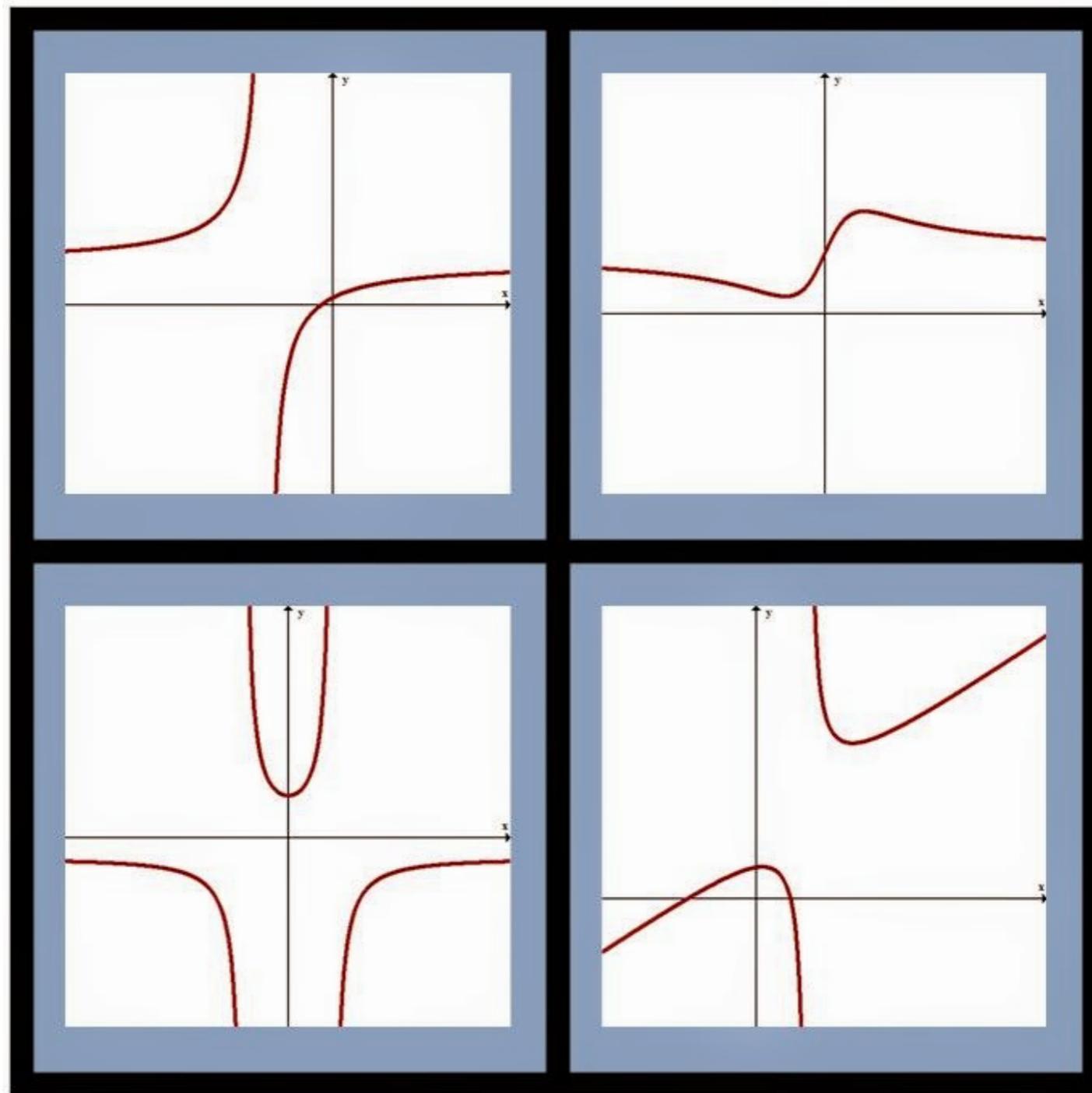
It can be all about the ideas, not the answer.

$$216 + 188$$

Ask About Ideas, Not Answers

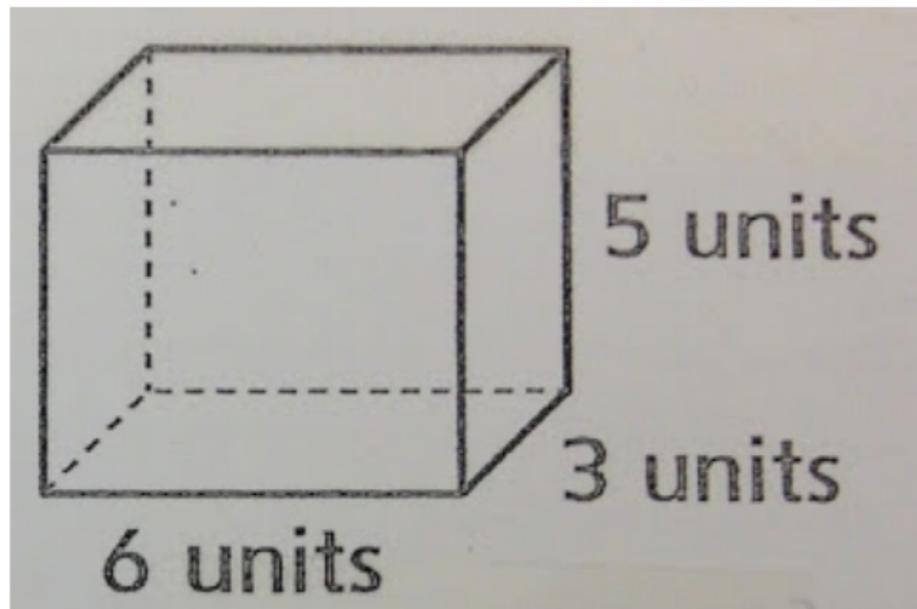


Ask About Ideas, Not Answers



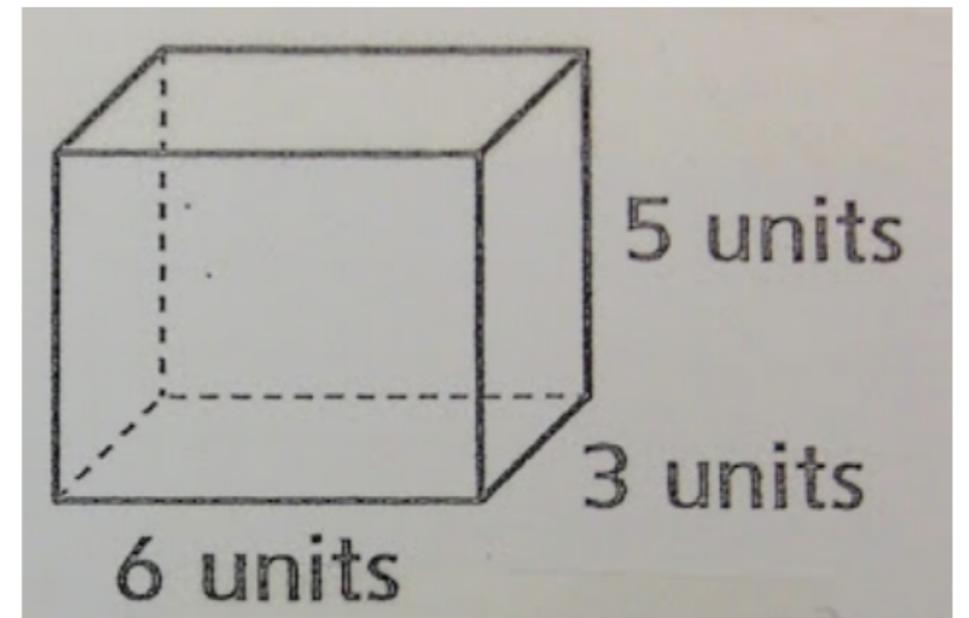
Ask About Ideas, Not Answers

It can be a little more complex:



Tell me everything you can about this figure.

instead of

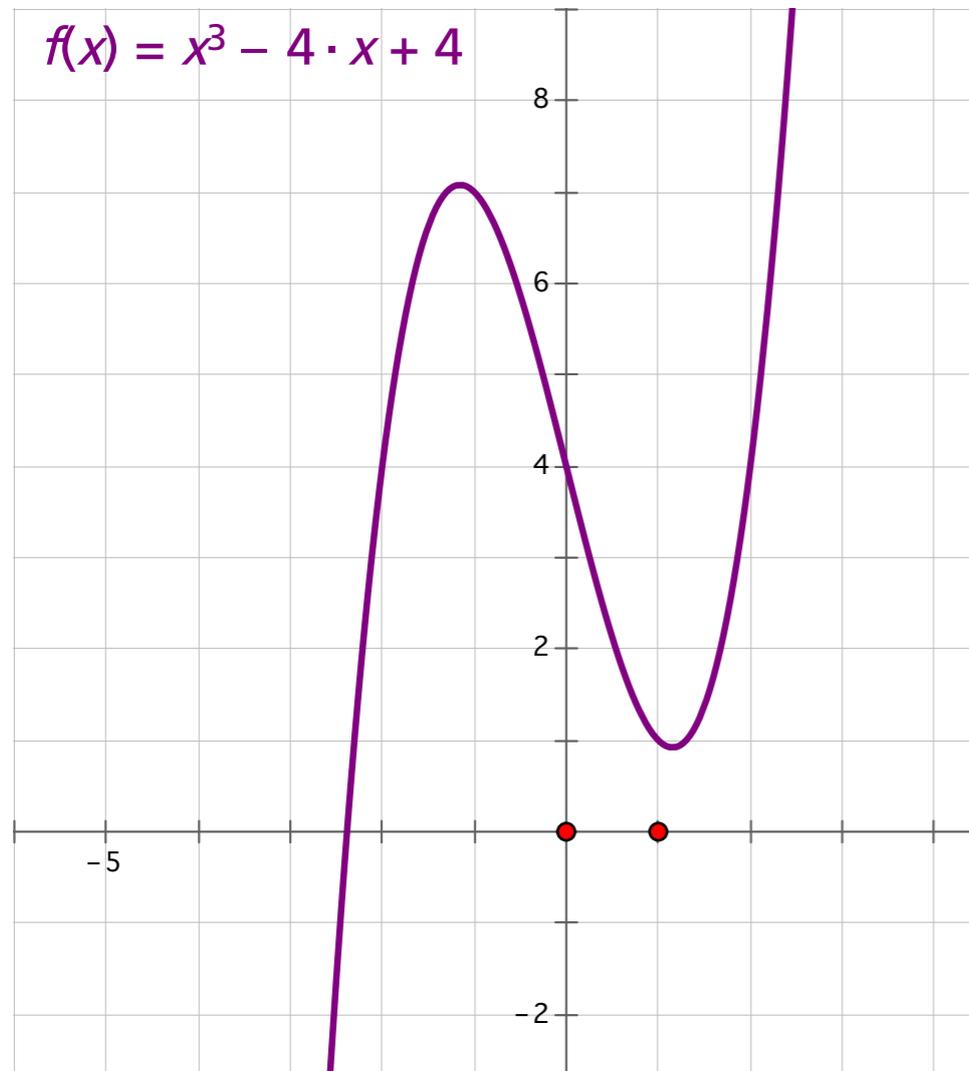


Find the volume of the rectangular prism.

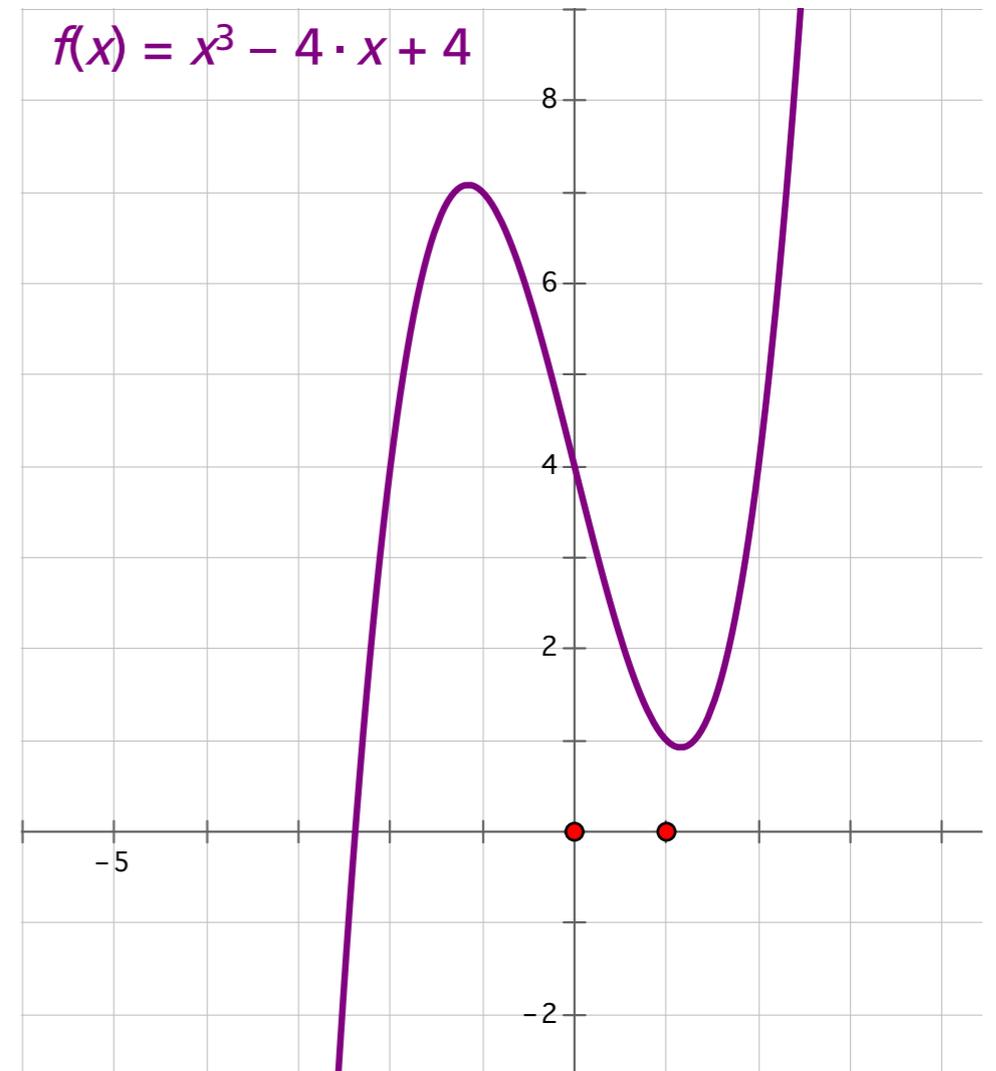
(from Joe Schwartz's blog, October 10, 2016)

Ask About Ideas, Not Answers

It can be even more complex:



instead of

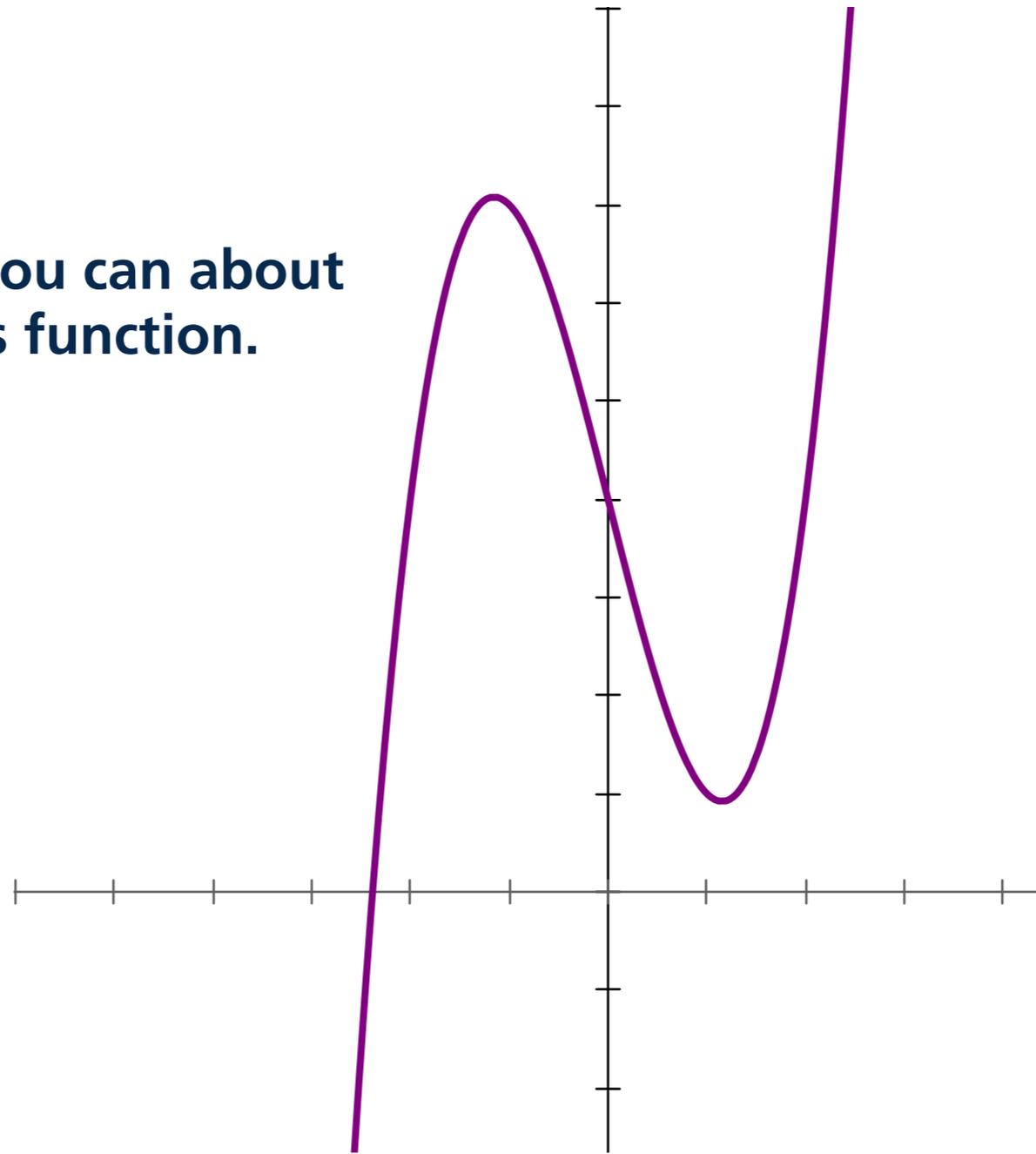


Explain everything you can about the derivative of this function.

Find the derivative of this function.

Ask About Ideas, Not Answers, and Without Numbers

Explain everything you can about
the derivative of this function.



Teacher Questions

“Why?”

“How do you know?”

“How did you decide?”

“Tell me more about that.”

“Phone in Pocket”

Are you asking *idea-focused* questions or *answer-focused* questions? Record yourself and find out!

#ToVForRatio

Ways to Encourage Sense Making Rather Than Answer Getting

- Get rid of the question.
- Get rid of the question and the numbers.
- Give the answer.
- Ask about ideas, not answers.

Mingle Questions

Pick One:

- How might one of these sense-making strategies be used with a specific task/routine you do in your classroom?
- How might you implement these ideas with the goal to get all students thinking and sharing?
- How could these ideas make core instruction more effective?
- What concerns do you have about implementing these ideas?

Everyone:

- What's something you're wondering about?

Mingle Questions

- Stand up and move around.
- Find someone and introduce yourself.
- Ask them one question from the list.
- Listen to their answer.
- Move on to find another person.
- No back and forth, just ask one question and listen to the answer.
- When I raise my hand, finish your conversation and raise your hand.

Mingle Reflections

Moment for Reflection and Personal (Possibly Public) Commitments

Thanks!

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