The Power of Students' Ideas

Annie Fetter

21st Century Partnership for STEM Education
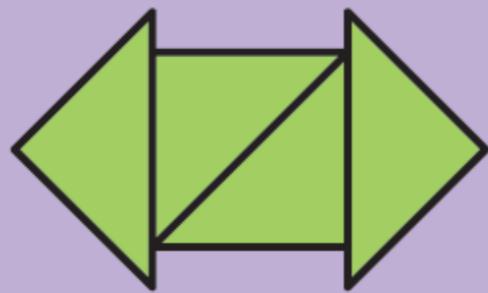@MFAnnie, anniefetter@gmail.com

CMC South 2021

Please sit with at least one other person (if you're COVID-okay with that).
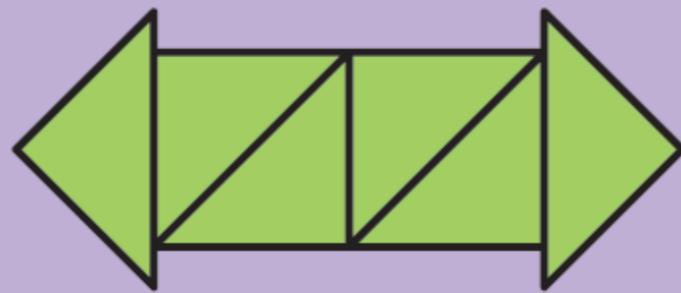
A PDF of the slides and pointers to some related resources will be available on my blog after the session: annie.mathematicalthinking.org
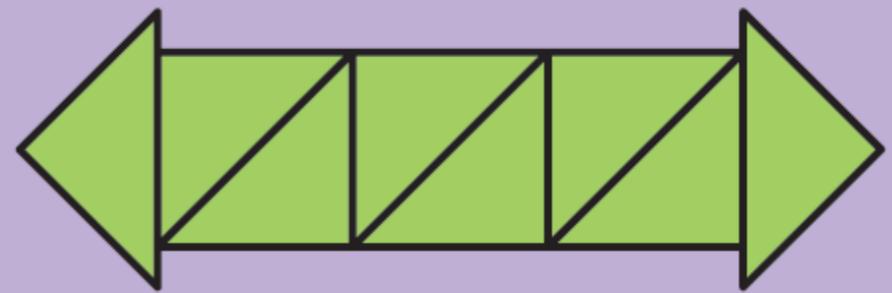
Annie Fetter
@MFAnnie
#NoticeWonder

Growing Worms Scenario

1-day worm   2-day worm   3-day worm

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Growing Worms Scenario

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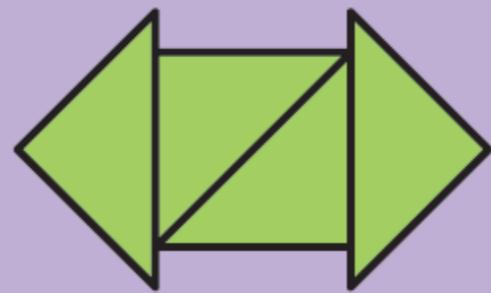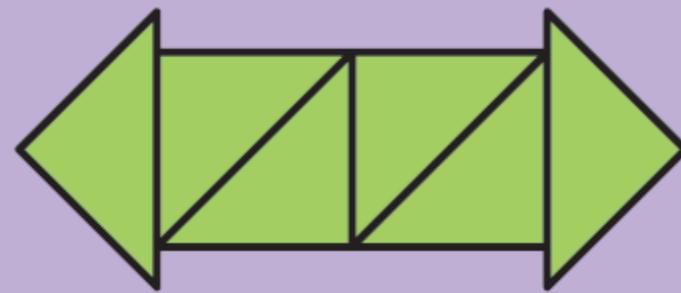
Growing Worms Scenario

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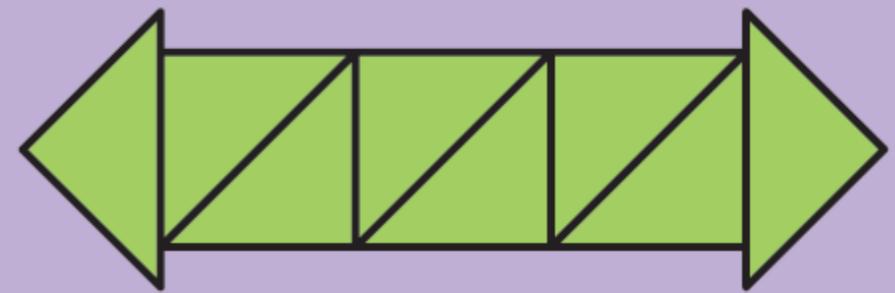# Growing Worms Scenario
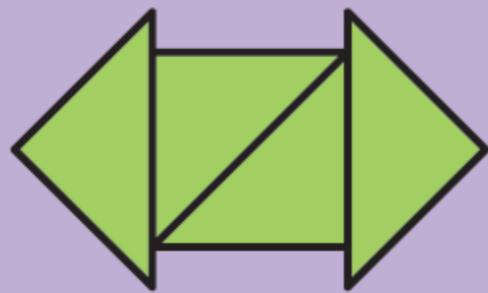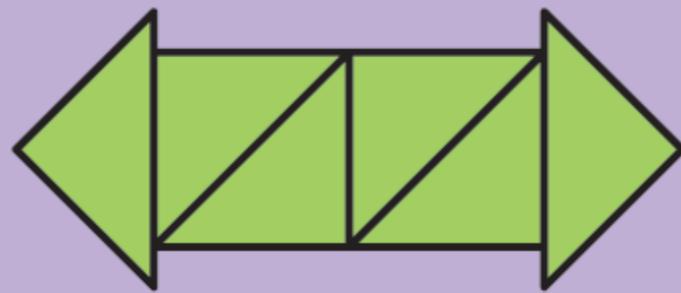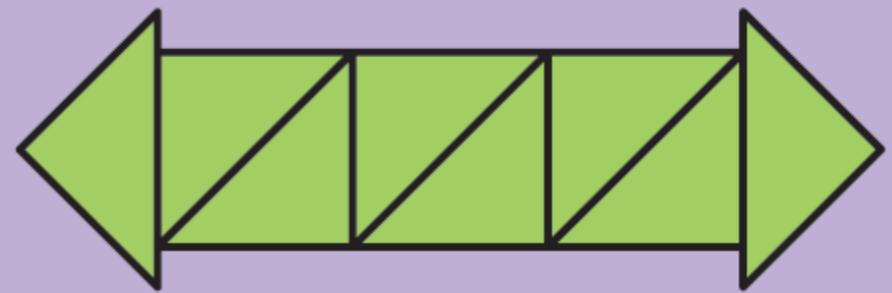
1-day worm

2-day worm

3-day worm

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I Notice | I Wonder

Growing Worms Movies

https://www.heinemann.com/pps/video.aspx

See, especially, the first three videos, where Val presents Growing Worms to 3rd graders.

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Growing Worms Student NW

N

- made of triangles
- adding by one cube (square) each day
- like a growing flower
- growing sideways like a worm
- more like a zigzag
- each step all even numbers
- 4, 6, 8 ... counting by 2s
- body of the worm is growing each day
- each day it gets longer
- green + black
- diagonal line through each square
- 2d shapes
- labels below each
- arrows on each end
- every day there's one more square

W

- is it a real worm?
- why is it going sideways instead of up
- what does this have to do with math?
- why is it made of triangles and not rectangles
- why isn't it 3D
- title growing worms?
- why are the shapes green?
- when it gets to 10 squares will it have a different shape
- when will the pattern stop
- why are arrows facing away?

Notice | Wonder

Notice

- that we made 3 worms and they are all different sizes
- we used different shapes - triangles and squares, too
- the worm gets bigger when we add a square
- everytime we made a new worm we added 1 square
- there was a pattern - 2 triangles, 3 squares
- it grew when we added a square
- triangle, square, triangle pattern

Wonder

- what the next worm will look like?
- if I could make
  - 1 million day
  - 5 day
  - 100 day
  - infinity
- if the worm can keep growing?
- if the worms could be a pet or if you could take it out to dinner with you?
- how cars are made?
- how triangles and squares are made?
- what would happen if the pattern would continue?

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How Many? How Did You Count?

Put your thumb up when you have an answer and are ready to describe how you figured it out.

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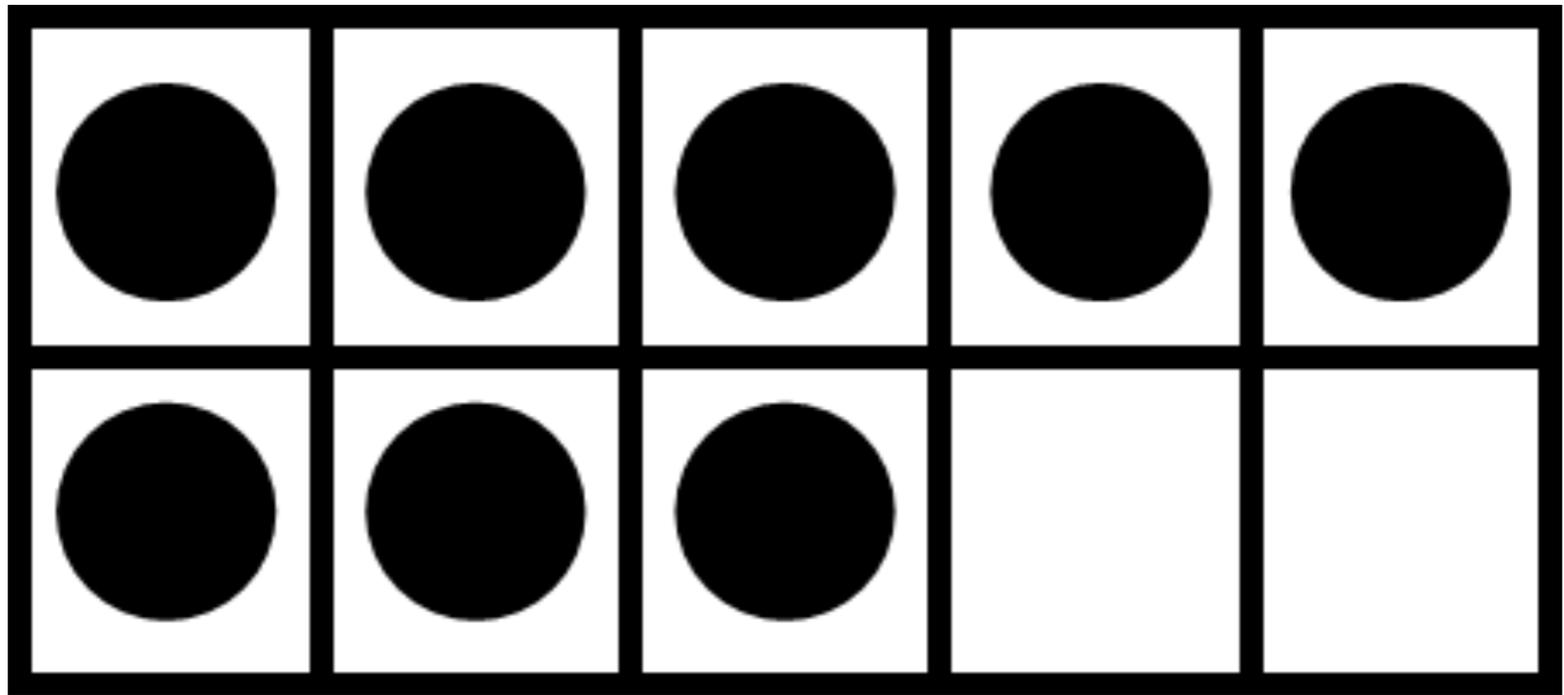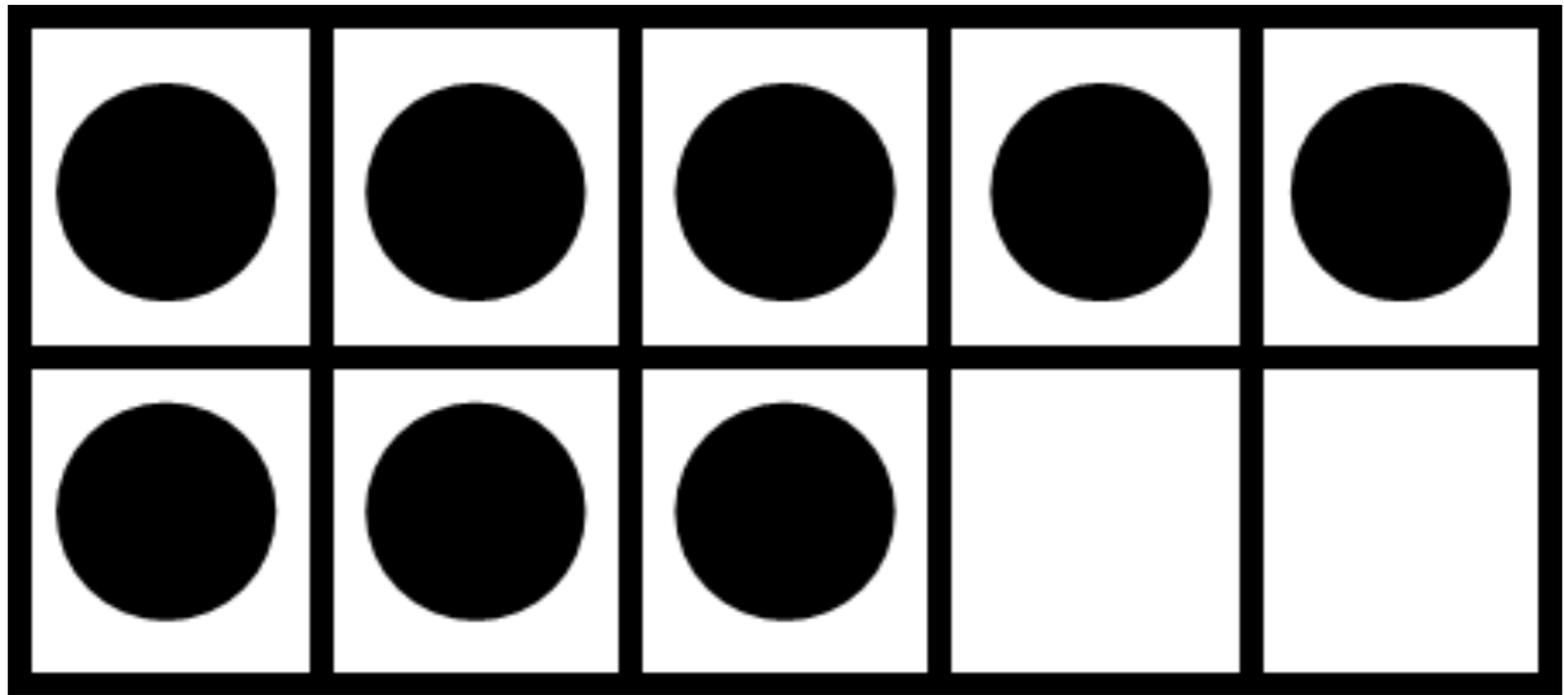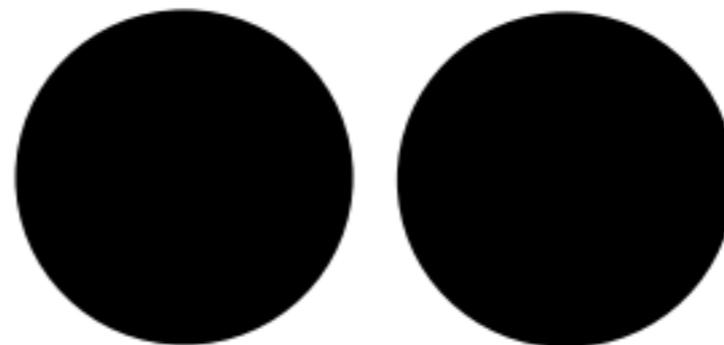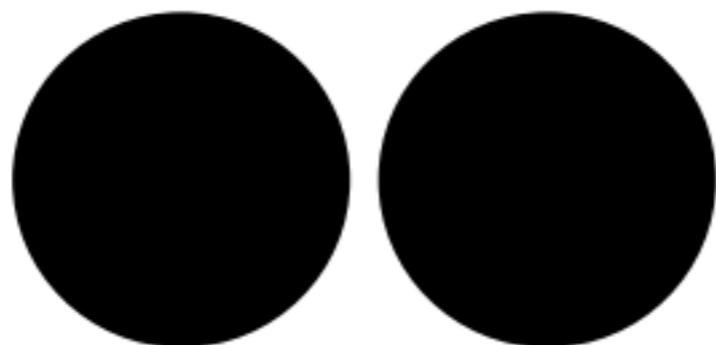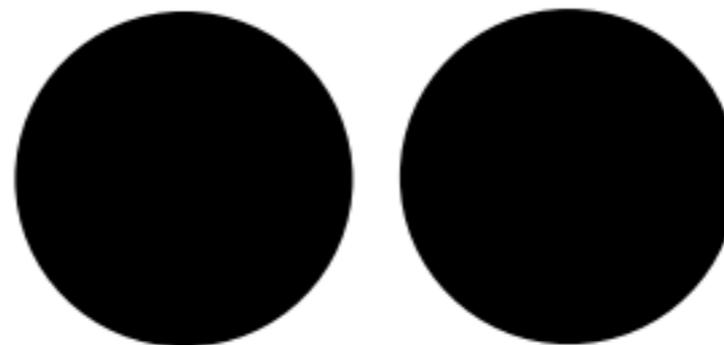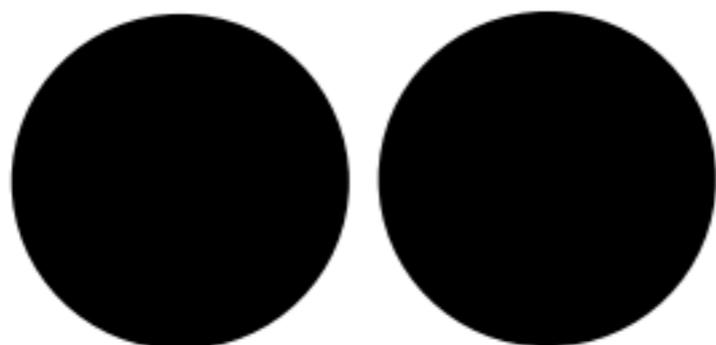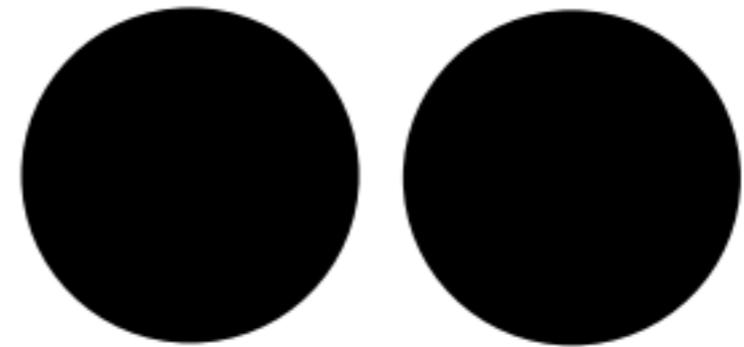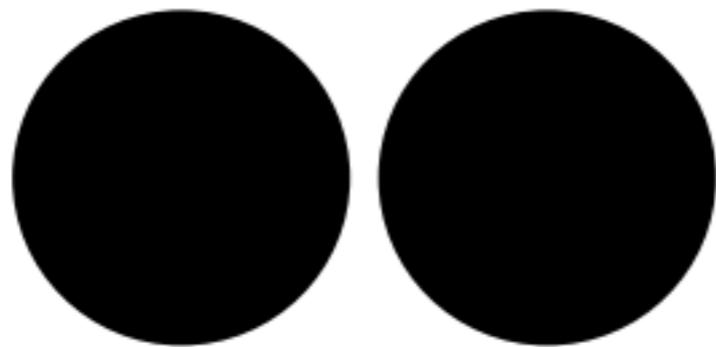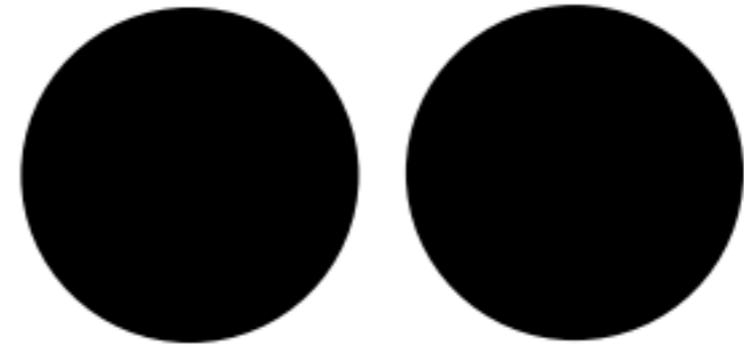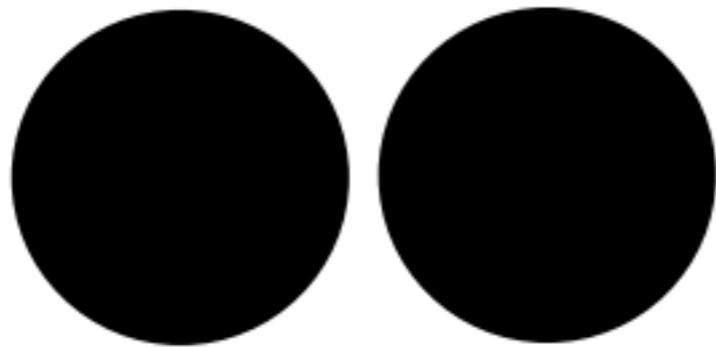
New Guidelines

- Put your thumb up when you have an answer and are ready to describe how you figured it out.
- Add another finger for every other way you see that it could be figured out.

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How Many? How Did You Count?

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15 + 16

26 + 49

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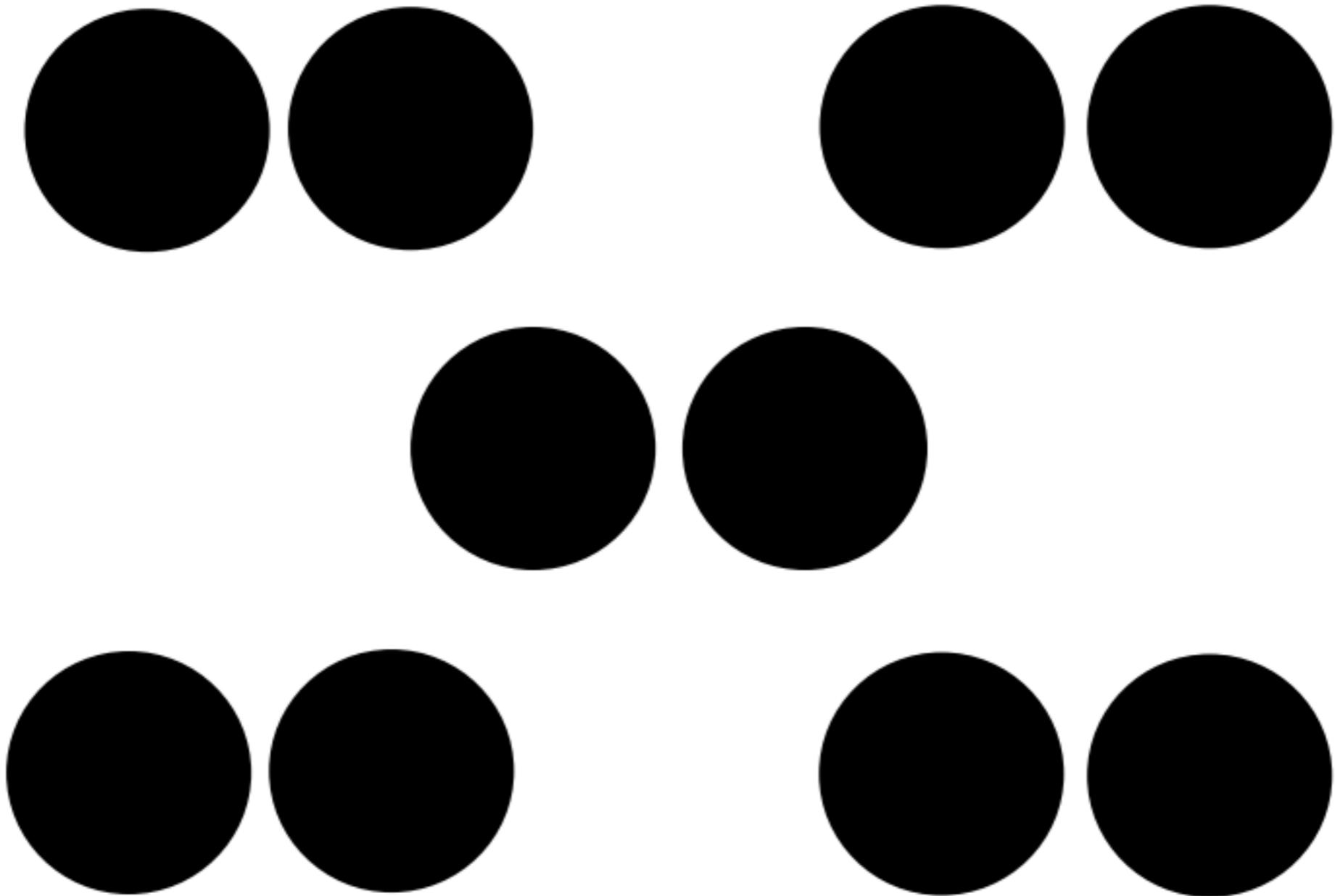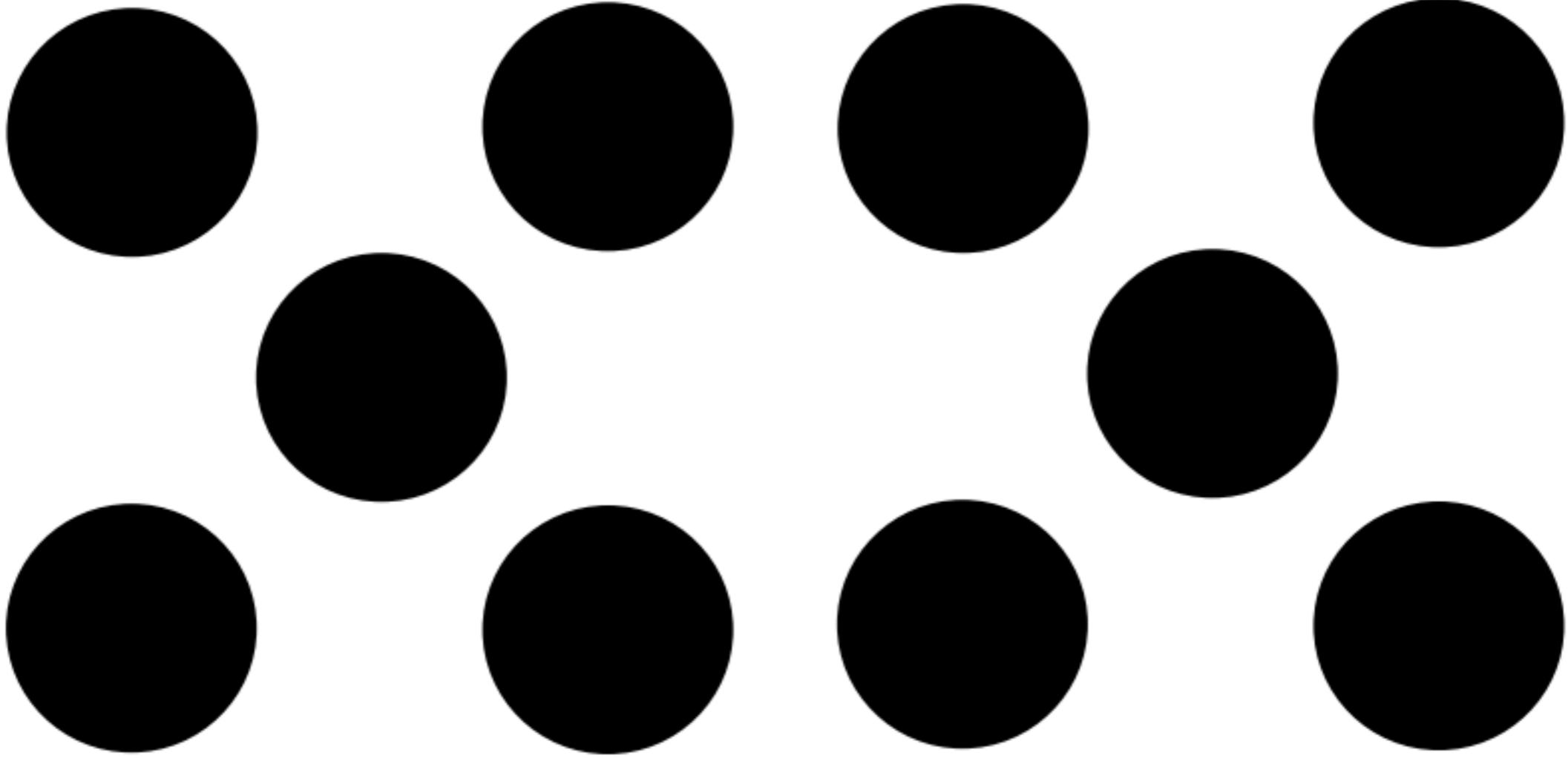
20 + 15

30 − 22

Number Talks

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Which One Doesn't Belong?

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Raul had some pet mice. Xavier gave him some

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice. How many mice did Raul have to start with?

A Numberless Word Problem from Brian Bushart, bstockus.wordpress.com

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Numberless Word Problems

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Routines

- How Many? How Did You Count?

- Number Talks

- Which One Doesn't Belong?

- Numberless Word Problems

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I used to think my job was to teach students to see what I see. I no longer believe this. My job is to teach students to see; and to recognize that no matter what the problem is, we don't all see things the same way. But when we examine our different ways of seeing, and look for the relationships involved, everyone sees more clearly; everyone understands more deeply.

—Ruth Parker

(Handwritten Notice and Wonder statements courtesy of student teachers I worked with at Swarthmore this fall.)

Wonder: Why do we start w/word problems when a lot of the kids don't know how to read yet?

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Be Curious

Some apples are on a tree.
A horse eats some apples.
Some apples are left on the tree.

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Notice: Students will blank/
disengage when the problem
seems hard.

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Characteristics of Strong ~~Readers~~ Mathematicians

- They are motivated to ~~read.~~ tackle problems
- They are able to ~~read words~~ recite facts accurately and automatically.
- They comprehend what they read.
- They are able to read with expression.
- They use a variety of strategies to tackle ~~words~~ problems they don't recognize.
- They use active problem solving strategies to search for information, to determine meaning, to make sense of words, to make connections.

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"Reading" Strategies – Unfamiliar Words

- Predicting
- Estimating
- Hypothesizing
- Make a movie in your mind
- Storyboarding (beginning, middle, end)
- Story elements (character, setting, problem, solution)

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CCSS Mathematical Practice 1

Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt.**

They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

They monitor and evaluate their progress and change course if necessary.

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Sample Grade 3 State Test Problem

The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?

A. 6   18%
B. 9   46%
C. 18   31%
D. 24   4%

Combined scores of the 160 third graders in a group of four low-performing schools I used to support.

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Sample Test Problem, Revised

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.

Sample Test Problem, Revised

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.

From Berkeley Everett's morning session, with kindergarteners doing 6 x 7:

"I know I have to show the story in a way that makes sense to me."

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Your Job:
Focus on and Monitor For Sense-Making Above All Else

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What happens when you focus on sense-making in kindergarten?

https://annie.mathematicalthinking.org/index.php/2017/09/20/emily-kindergartners-and-sense-making/

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Notice: Kids still focusing on the answer even tho we're pushing for math teaching.

Wonder: How to change students' perspectives on math.

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YouCubed at Stanford

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Encouraging Sense Making

Q: What's one way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question. Literally.

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Get Rid of the Question

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

I Notice	I Wonder

Get Rid of the Question

▶ Relate Pictures to Tens and Ones **MATH TALK**

MP.1 Make Sense of Problems Analyze the Problem Discuss the pictures in Exercises 1 and 2. Count the number of cars in the first row. **10 cars** Explain that drivers may be directed to fill a row before parking in the next row of a parking lot. In the same way, people may be asked to fill a row of seats before sitting in the next row at a theater.

- How can a filled row help you count the number of cars or the number of people? **Possible response: A filled row shows ten, so I can use the picture to count tens and extras.**

- How do the cars in Exercise 1 show tens and ones? **2 filled rows show tens and 3 extra cars show ones.**

- How do the people in Exercise 2 show tens and ones? **There are 4 rows of ten with 6 extra ones. This time the ones are at the top and the tens are shown below.**

4-18 Class Activity

Name _____

▶ **Math and the Community Theater**

Linda and her family go to a show.

1. 10 cars can park in each row.
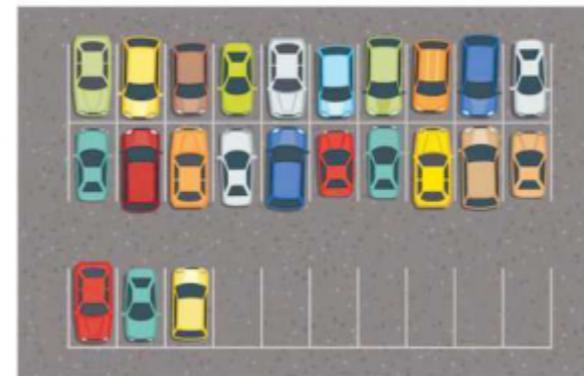
How many cars are there?

_____ tens _____ ones = _____ cars

2. 10 people can sit in each row.

How many people are there?

_____ tens _____ ones = _____ people

Get Rid of the Question

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Things 5th Graders Say about NW-ing

"…it helps me see new things I wouldn't have seen."

"…there are multiple answers so you can't really be wrong with it."

"…helps me look at a problem in a way I never thought of."

"…you get to think about the problem more and you realize more."

"…we don't have to do math at all, we just need to think on it without stress."

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"You get to share your
own thinking and no one
can ruin it."

—Aya, Grade 2

Ask for Questions, Not Answers

Solve & Share

493

A pet store has 162 goldfish, 124 angelfish, and 53 pufferfish. How many fish are there in all? How might an estimate help you solve the problem? Solve this problem any way you choose.

0:01:44

When you are finished, turn to page 497, circle numbers 1-8, 9, 12, and 13, carefully rip it out and put it in your take-home folder.

If you finish early complete the Look Back.

Thursday

A pet store has 162 goldfish 124 angelfish, and 53 pufferfish. What questions could you come up with?

00:03 39

5 minutes on your own

3 with a partner

All together

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Ask for Questions, Not Answers

pufferfish. What questions could you come up with?

How much more does the goldfish's has more than the angelfish?

What is the total of all fishes?

How much does the putterfish and the gold fish have altogether?

How much does the angelfish and

What could the questions be now?

how much more is blah blah blah than blah blah blah?

Encouraging Sense-Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question *and* the numbers.

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#NoticeWonder

Get Rid of the Question
and the Numbers

Raul had some pet mice. Xavier gave him some more mice.

Raul had some pet mice. Xavier gave him 3 more mice.

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice.

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice. How many mice did Raul have to start with?

A Numberless Word Problem from Brian Bushart, bstockus.wordpress.com

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Get Rid of the Question and the Numbers

15. The area of the rectangle is ▓▓▓▓▓▓▓▓▓ One side of the rectangle has a length of 10 meters.
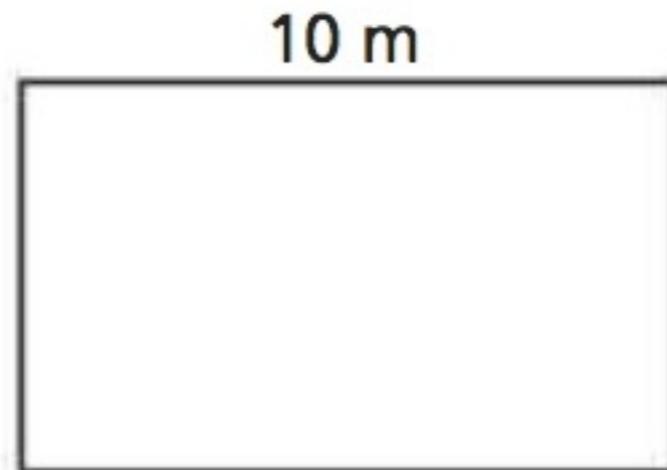
10 m

Get Rid of the Question and the Numbers

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Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Give the answer and let the students do the work.

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Give the Answer

◆ Math Message Follow-Up

WHOLE-CLASS ACTIVITY

Draw or display a function machine and "What's My Rule?" table. (See Advance Preparation.)

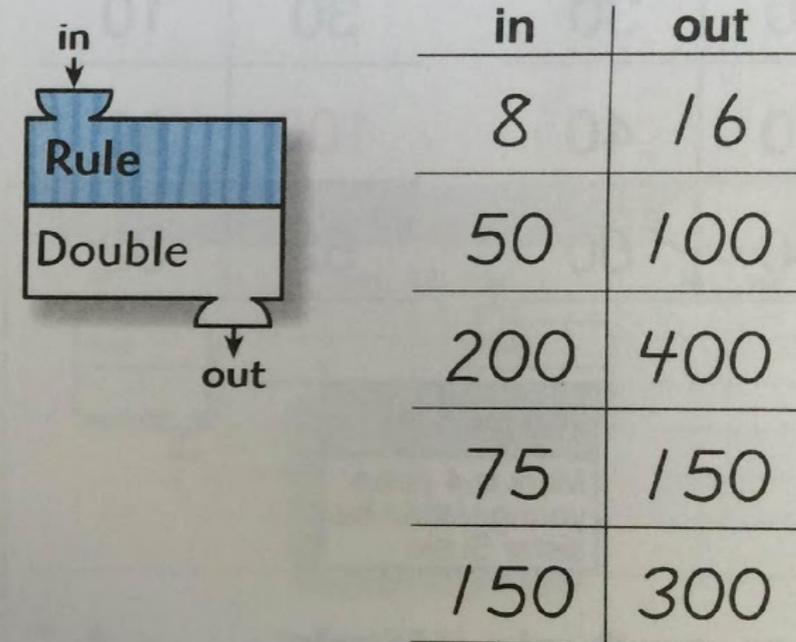Ask children to imagine that the **function machine** works like this:

- A number (the **input**) is dropped into the machine,
- the machine changes the number according to a rule,
- and a new number (the **output**) comes out the other end.

The **rule** for the Math Message problem is "Double the number." Write the word *Double* in the function machine.
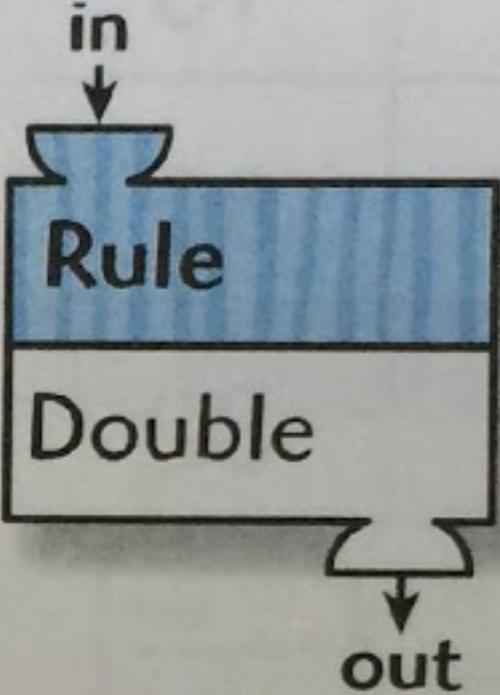
Point out the **"What's My Rule?"** table. Discuss the 8 in the *in* column and the 16 in the *out* column. Explain to children that numbers in the *in* column represent the numbers of bacteria now. Corresponding numbers in the *out* column represent the numbers of bacteria 20 minutes from now.

Review the answers to the Math Message problem by posing questions in the following manner:

- If 50 is dropped into the function machine, which number will come out? 100 Enter the appropriate numbers in the *in* and *out* columns.

in →

Rule

Double

→ out

in	out
8	16
50	100
200	400
75	150
150	300

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Give the Answer

in	out
8	16
50	100
200	400
75	150
150	300

Give the Answer (or Several!)

Rachel bakes cookies and delivers them to her friends.

- It takes 8 minutes to mix the batter.
- The cookies bake for 9 minutes.
- For 6 minutes they cool.

If the answer is 23 minutes, what is the question?

If the answer is 3 minutes, what is the question?

If the answer is bake, what is the question?

Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Ask about ideas, not answers.

This can be really simple:
"Tell me something about number 7."
instead of
"What's the answer to number 7?"

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Ask About Ideas, Not Answers

It can be a little more complex:

5 units

3 units

6 units
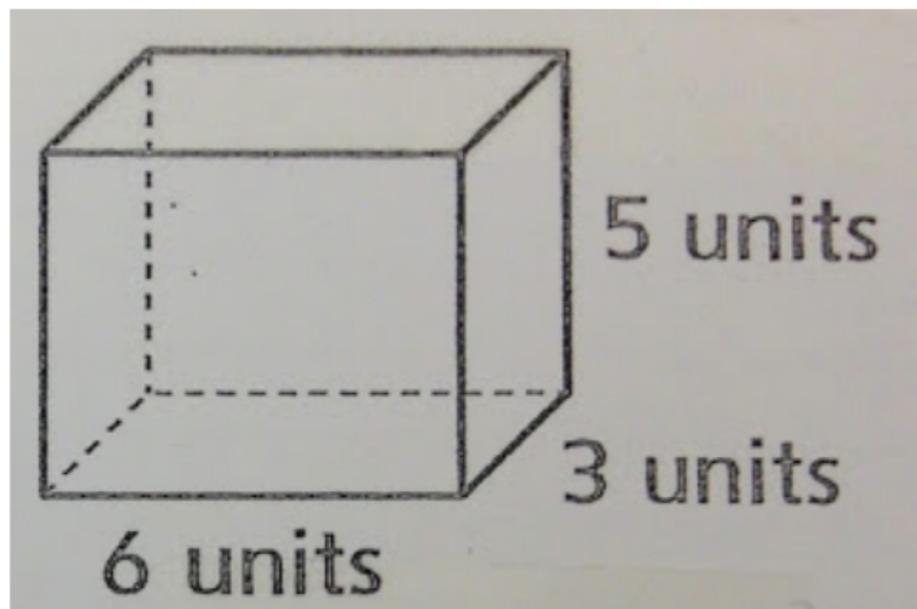
Tell me everything you can about this figure.

instead of

5 units

3 units

6 units
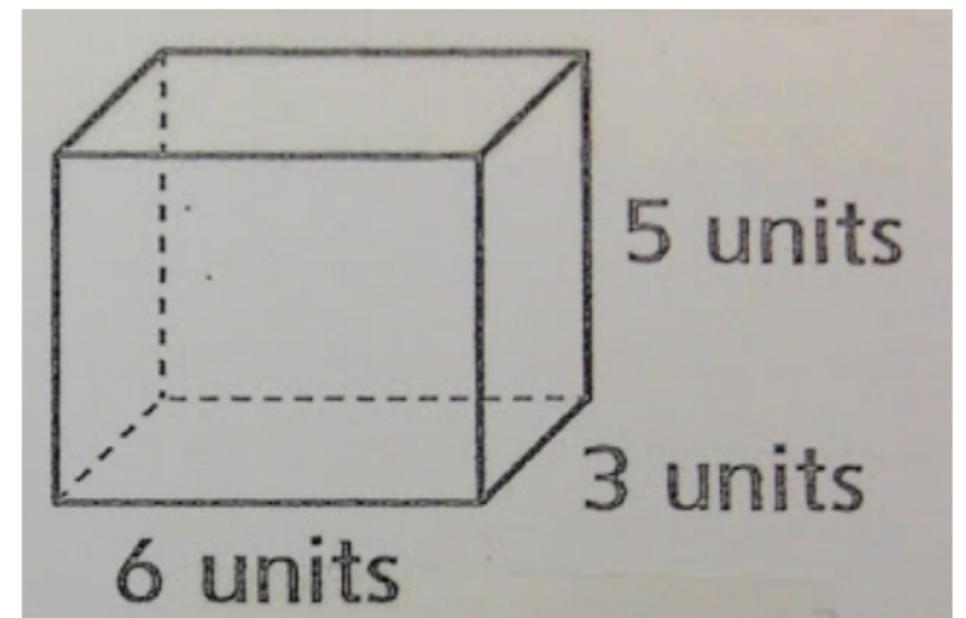
Find the volume of the rectangular prism.

(from Joe Schwartz's blog, October 10, 2016)

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Ask About Ideas, Not Answers

1. Suppose 5 U.S. dollars (5 USD) can be exchanged for 64 Mexican pesos. What operation would be used to find the value of 1 USD in pesos?

   division

   Find the value of 1 USD in pesos. 1 USD = ___12.8___ pesos

Tell everything you can about this statement: 5 U.S. dollars (5 USD) can be exchanged for 64 Mexican pesos.

Teacher Questions

"Why?"

"How do you know?"

"How did you decide?"

"Tell me more about that."

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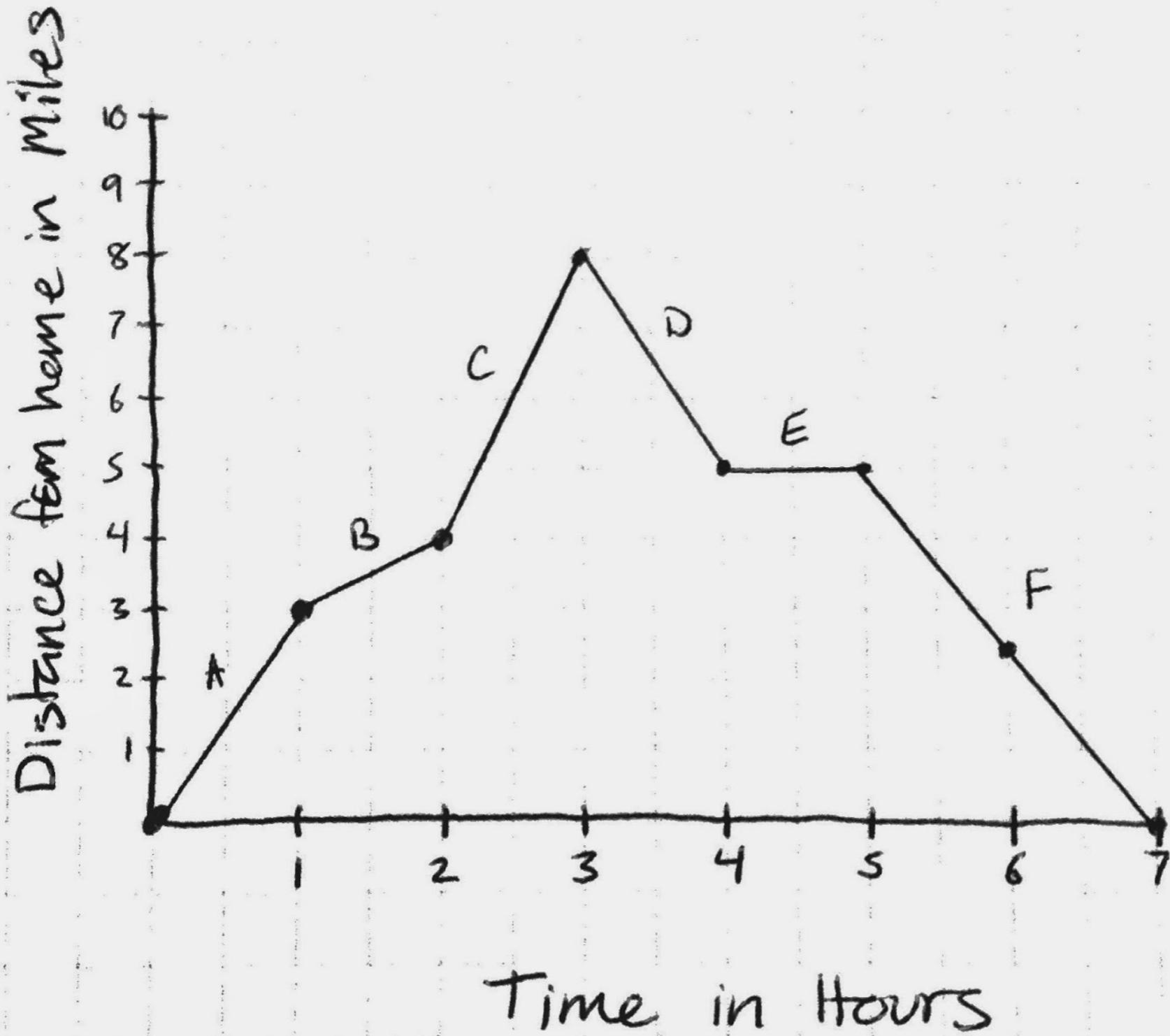@MFAnnie
#NoticeWonder

Ways to Encourage Sense Making Rather Than Answer Getting

- Get rid of the question.

- Get rid of the question *and* the numbers.

- Give the answer.

- Ask about ideas, not answers.

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Wonders:

How do you do this in a time crunch?

How to adapt to mandated scope and sequence when students aren't "ready".

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#NoticeWonder

Tina Cardone 🏳️‍🌈
@TinaCardone

• • •

Replying to @MFAnnie

@MFAnnie when I gave the graph and did notice/wonder first I didn't have to answer nearly so many questions when they did the handout

5:36 PM · Nov 24, 2014 · Tweetbot for iOS

Replying to @MFAnnie

@MFAnnie worth the few minutes it took and meant we skipped wrap up discussion (they already had it)
drawingonmath.blogspot.com/2014/11/distan...

5:37 PM · Nov 24, 2014 · Tweetbot for iOS

http://drawingonmath.blogspot.com/2014/11/distance-graph.html

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As young teachers, we believed our job was to carefully explain what we knew about mathematics to our students. We asked questions and listened to our students' answers but our listening was aimed at assessing whether our students got what we had explained rather than uncovering their understanding of the content.

We now see that we missed valuable opportunities to develop students' understanding because we did not elicit their ideas or relate their ideas to the content we were teaching.

—Susan B. Empson and Linda Levi
Extending Children's Mathematics: Fractions and Decimals

anniefetter@gmail.com

Annie Fetter
@MFAnnie
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