

# The Power of Ideas

Annie Fetter, @MFAnnie, anniefetter@gmail.com

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A PDF of the slides will be available after the session at  
[annie.mathematicalthinking.org](http://annie.mathematicalthinking.org)

Links to all the technology will eventually be available as well.

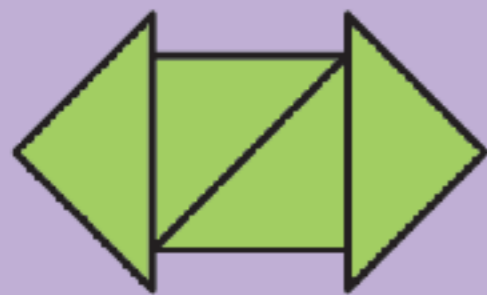
## We notice ...

- Kindergarteners point and verbally count
- Out of order numerical sequence confuses K
- Numberline is not visibly placed
- Ss have difficulty conceptualizing numbers in abstract form, need and object to imagine (e.g. apples, chocolate)
- Wide range of ability/experience
- instruction feels tethered to "the book"
- a lack of found. math facts  $\rightarrow$  only count

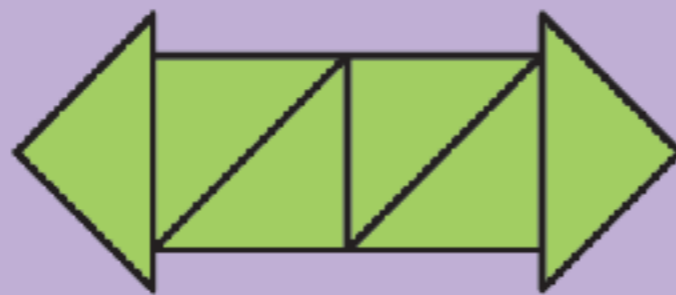
## We wonder...

- where do rekenreks come from?
- steps from concrete  $\rightarrow$  abstract  
visual tools  $\rightarrow$  just calculators?
- 1-to-1 typical K instruction?
- how do you embrace a diversity of approaches (to arithmetic, for example), while still lifting up the methods/approaches that are the most efficient  
side Q: should efficiency even be a point of concern?
- how to create an explanatory math environment in a constraining administrative/policy context?

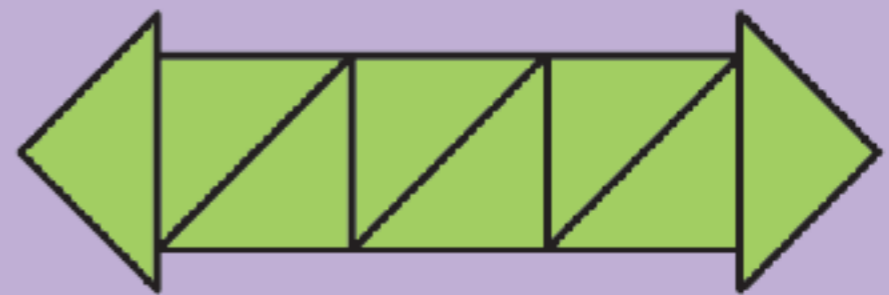
# Growing Worms Scenario



1-day worm



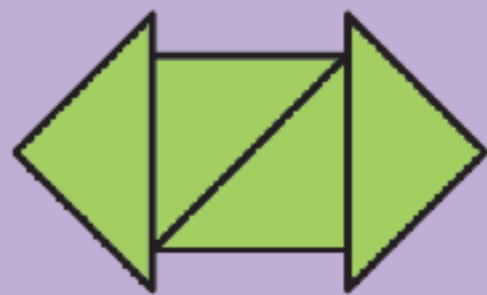
2-day worm



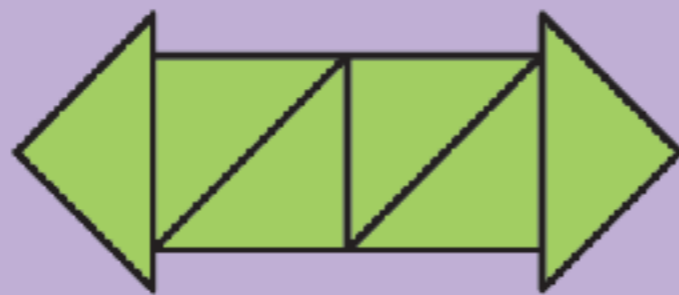
3-day worm

# Growing Worms Scenario

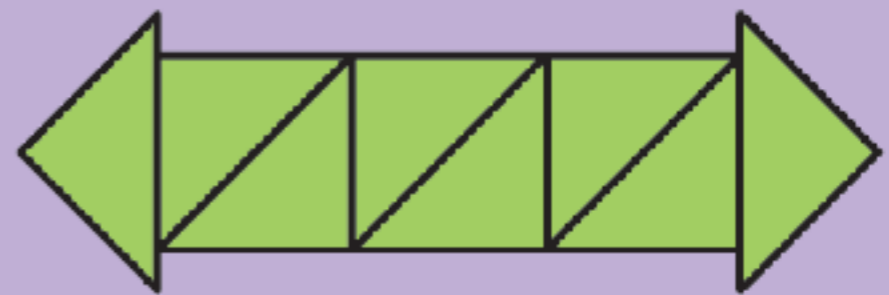
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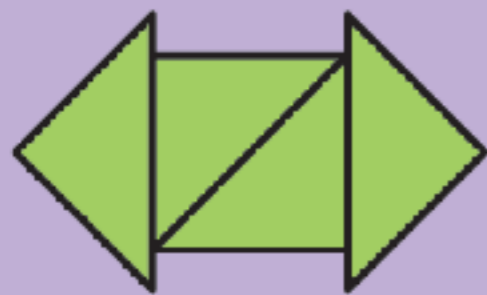
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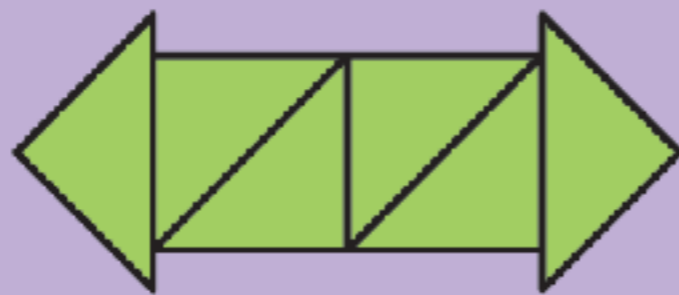
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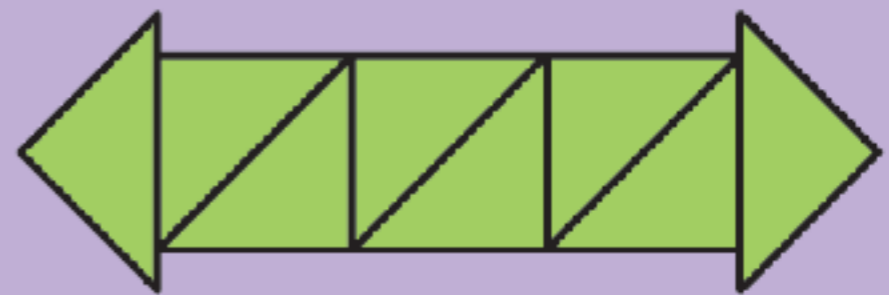
# Growing Worms Scenario



1-day worm



2-day worm



3-day worm

**I Notice**

**I Wonder**



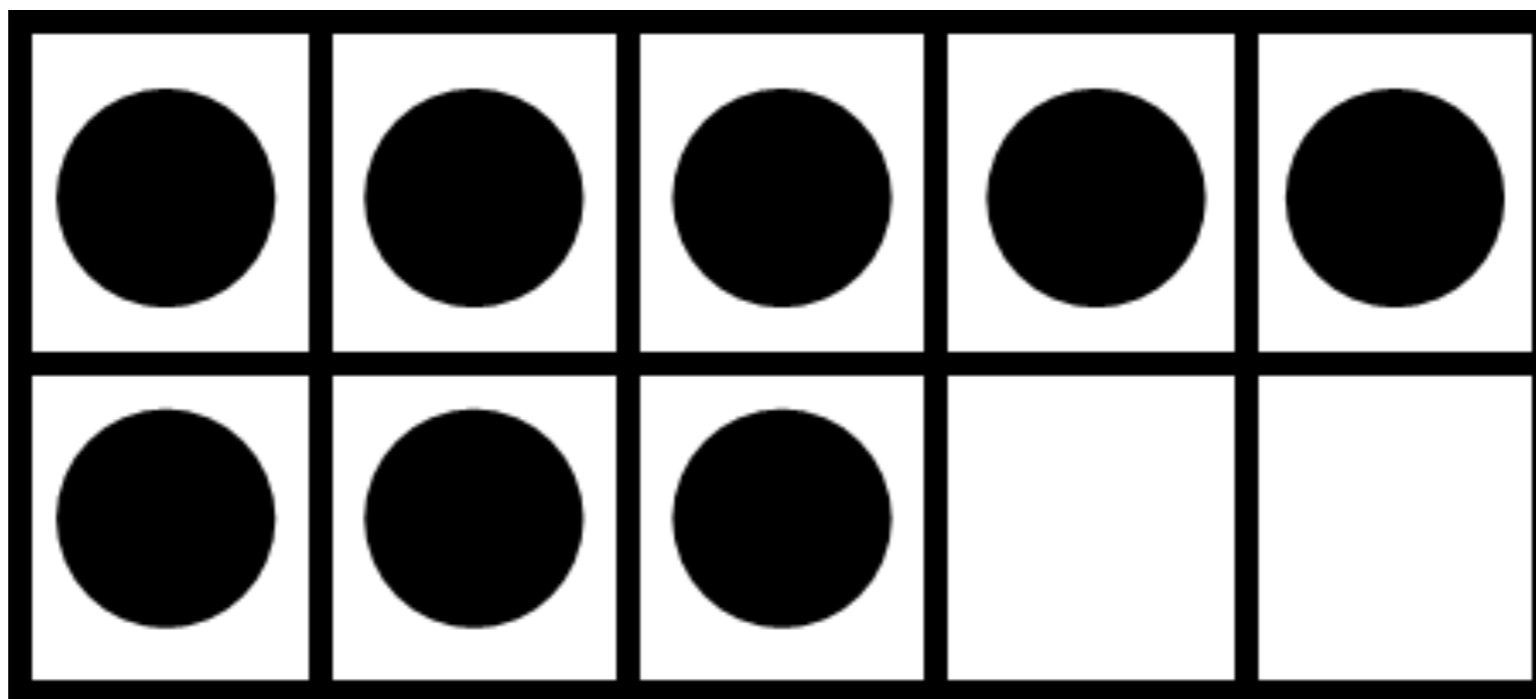
# Growing Worms Student NW

- | N   | W   |
|---|---|
| - made of triangles                           | - is it a real worm?  |
| - adding by one cube (square) each day        | - why is it going sideways instead of up                    |
| - like a growing flower                       | - what does this have to do with math?                      |
| - growing sideways like a worm                | - why is it made of triangles and not rectangles            |
| - more like a zigzag                          | - why isn't it 3D   |
| - each step all even numbers                  | - title growing worms?                                      |
| - 4, 6, 8... counting by 2s                   | - why are the shapes green?                                 |
| - <u>body</u> of the worm is growing each day | - when it gets to 10 squares will it have a different shape |
| - each day it gets longer                     | - when will the pattern stop                                |
| - green + black                               | - why are arrows facing away?                               |
| - diagonal line through each square           |   |
| - 2d shapes                                   |   |
| - labels below each                           |   |
| - arrows on each end                          |   |
| - every day there's one more square           |   |

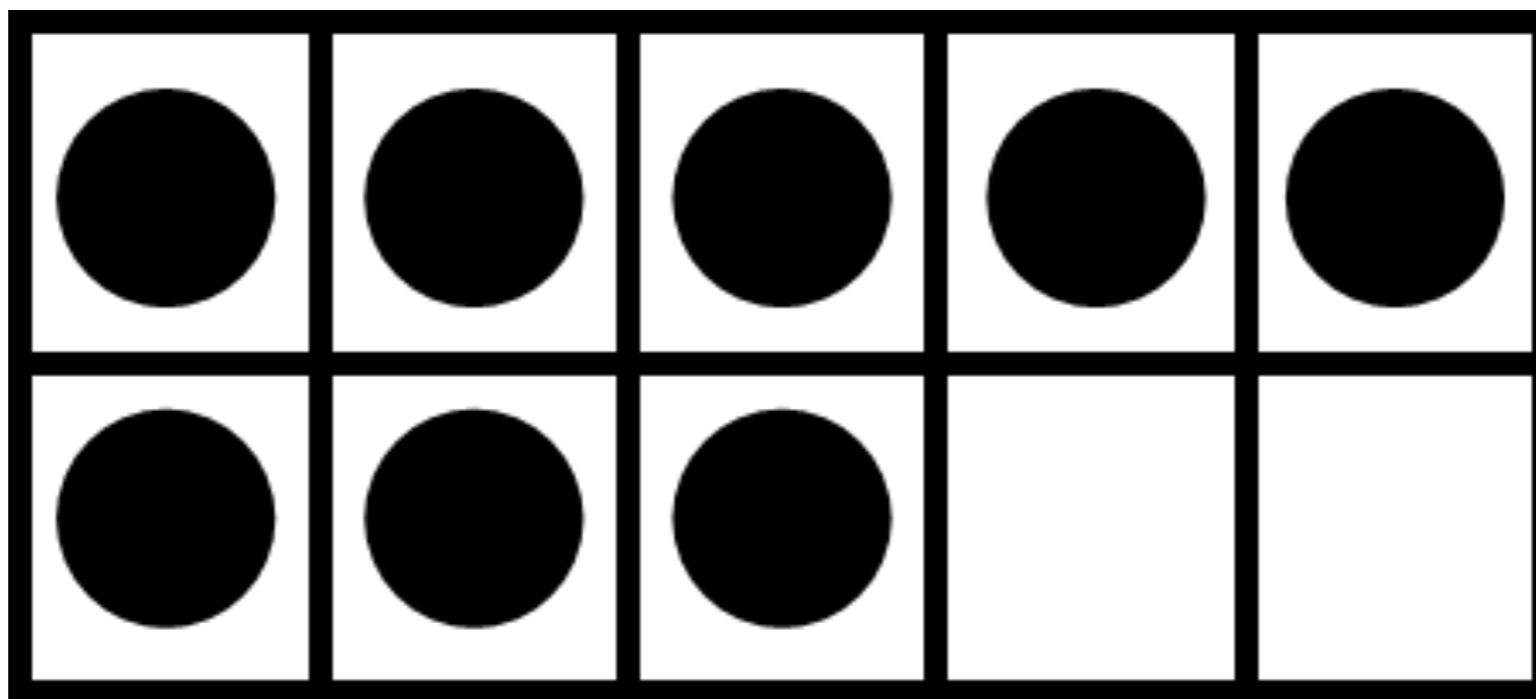
- | Notice  | Wonder  |
|---|---|
| - that we made 3 worms and they are all different sizes | - what the next worm will look like?  |
| - we used different shapes - triangles and squares, too | - if I could make <ul style="list-style-type: none"> <li>· 1 million day</li> <li>· 5 day</li> <li>· 100 day</li> <li>· infinity</li> </ul> |
| - the worm gets bigger when we add a square             | - if the worm can keep growing?   |
| - everytime we made a new worm we added 1 square        | - if the worms could be a pet or if you could take it out to dinner with you?   |
| - there was a pattern - 2 triangles, 3 squares          | - how cars are made?  |
| - it grew when we added a square                        | - how triangles and squares are made?   |
| - triangle, square, triangle pattern                    | - what would happen if the pattern would continue?  |

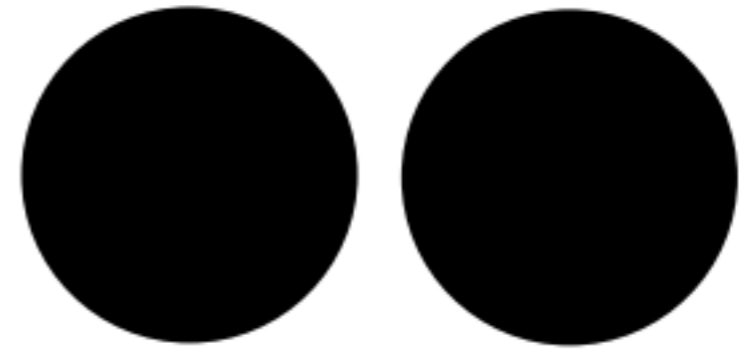
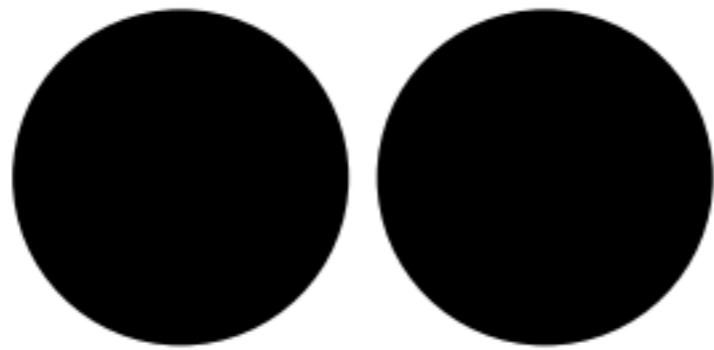
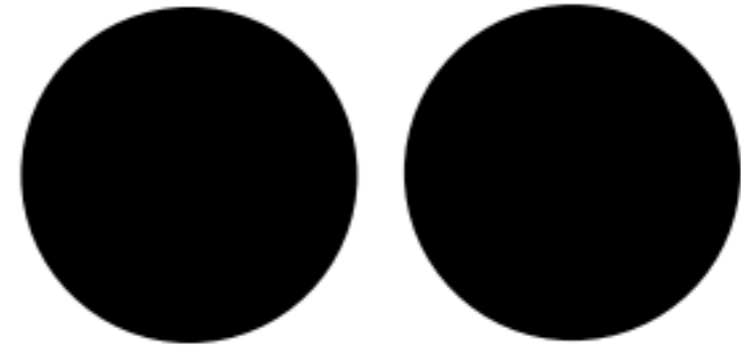
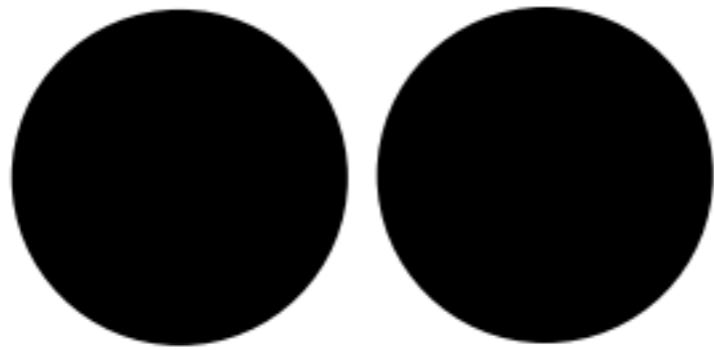
# Let's Count!

Put your thumb up when you have an answer and are ready to describe how you figured it out.

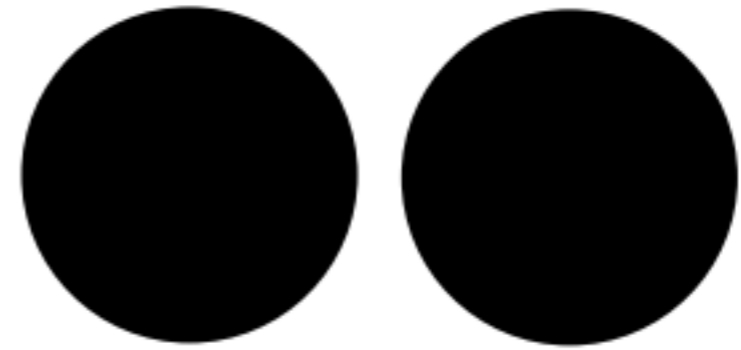
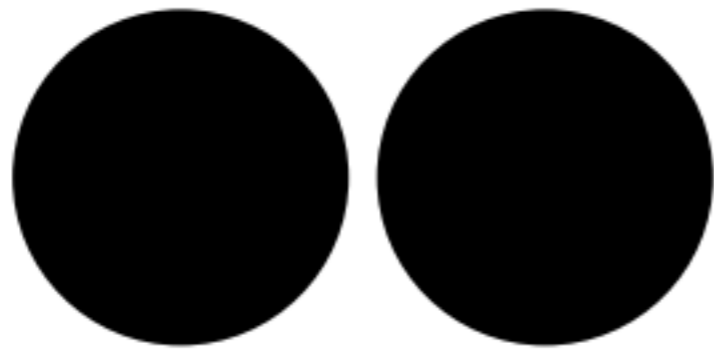
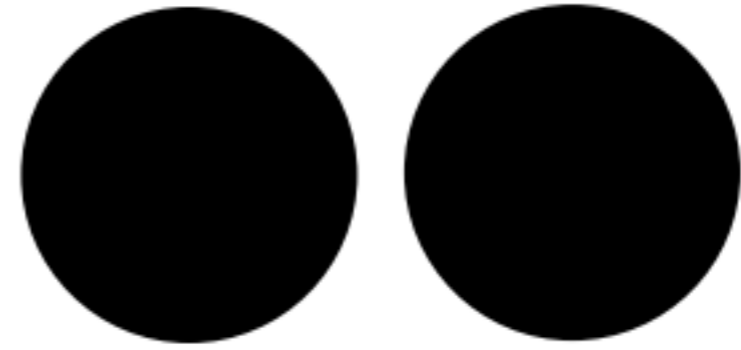
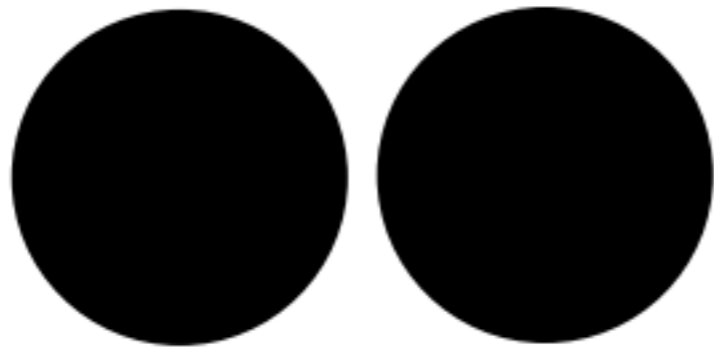








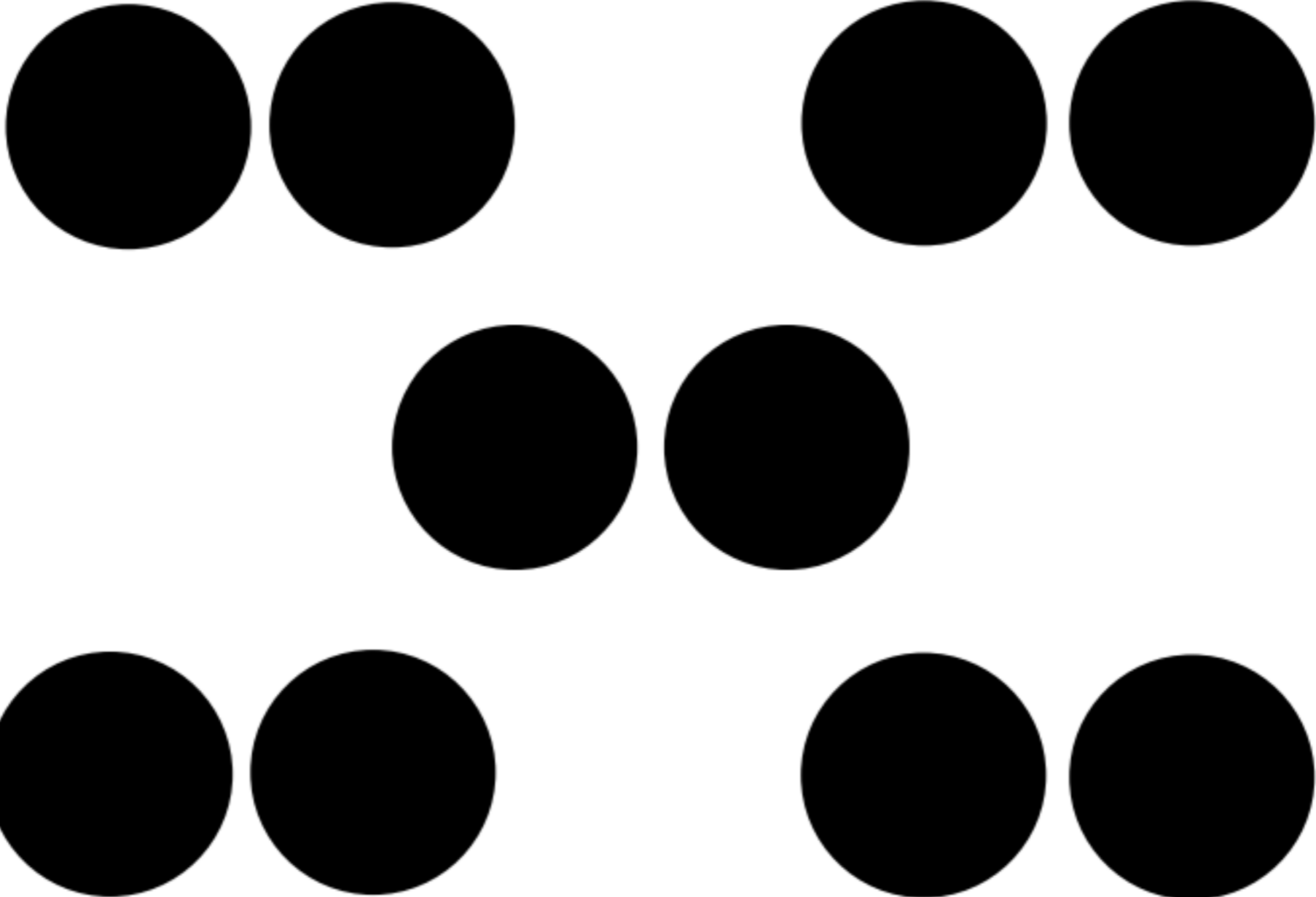






# New Guidelines

- Put your thumb up when you have an answer and are ready to describe how you figured it out.
- Add another finger for every other way you see that it could be figured out.



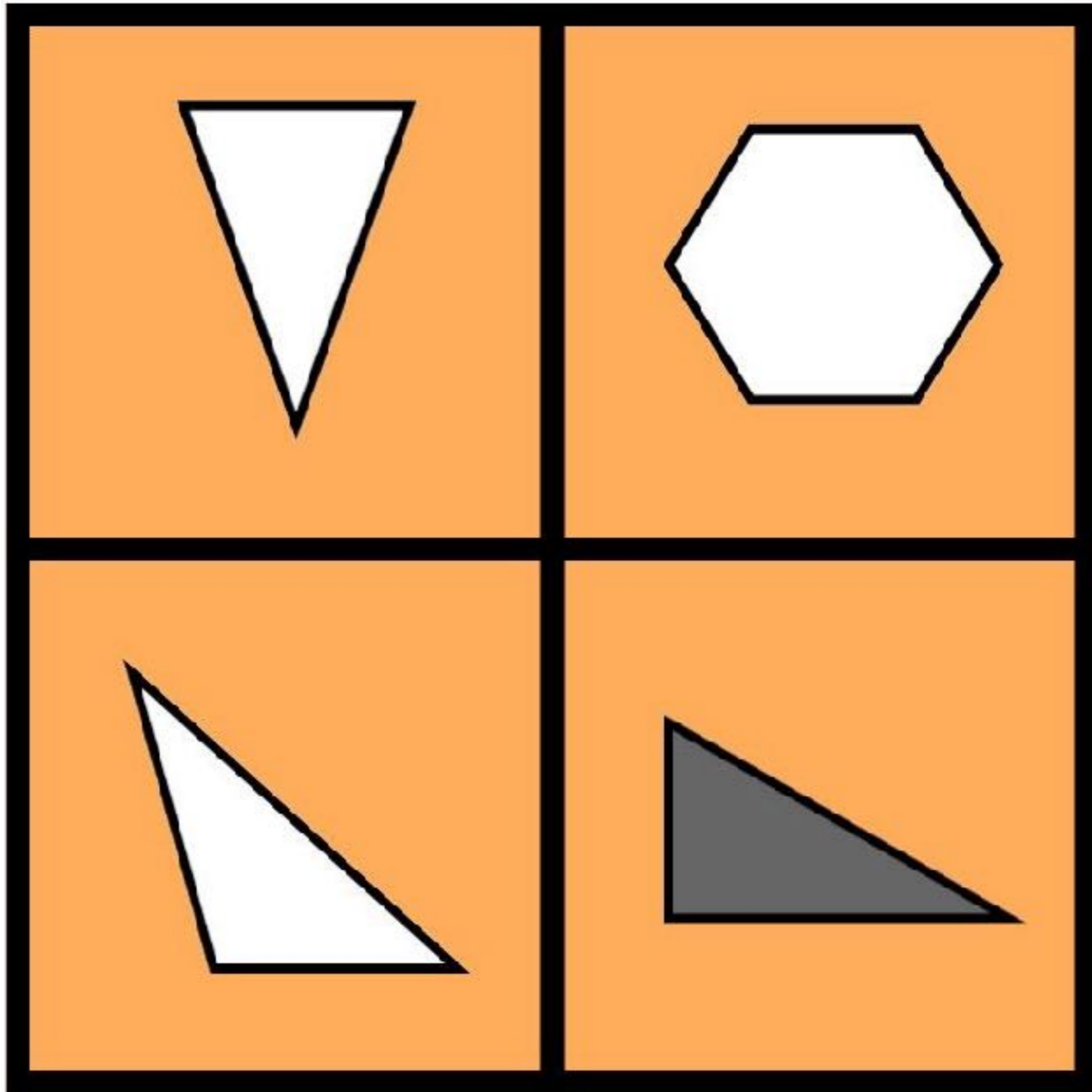
# How Many? How Did You Count?

**15 + 16**

26 + 49

# Number Talks

9	16
25	43





# Which One Doesn't Belong?

Raul had some pet mice. Xavier gave him some more mice. Now Raul has 8 mice.

~~Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice.~~

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice. How many mice did Raul have to start with?

*A Numberless Word Problem from Brian Bushart, [bstockus.wordpress.com](http://bstockus.wordpress.com)*

# Numberless Word Problems

# Routines

- How Many? How Did You Count?
- Number Talks
- Which One Doesn't Belong?
- Numberless Word Problems

I used to think my job was to teach students to see what I see. I no longer believe this. My job is to teach students to see; and to recognize that no matter what the problem is, we don't all see things the same way. But when we examine our different ways of seeing, and look for the relationships involved, everyone sees more clearly; everyone understands more deeply.

—Ruth Parker

# Grade 3 State Test Problem

The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?

- A. 6    18%
- B. 9    46%
- C. 18   31%
- D. 24   4%

Combined scores of the 160 third graders in a group of four low-performing schools I used to support.

# Sample Test Question, Revised

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.



# CCSS Mathematical Practice 1

**Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt.**

They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

They monitor and evaluate their progress and change course if necessary.



# Characteristics of Strong ~~Readers~~ Mathematicians

- They are motivated to ~~read~~. **tackle problems**
- They are able to ~~read words~~ accurately and automatically. **recite facts**
- They comprehend what they read.
- They are able to read with expression.
- They use a variety of strategies to tackle ~~words~~ **problems** they don't recognize.
- They use active problem solving strategies to search for information, to determine meaning, to make sense of words, to make connections.

# Reading Lesson Objectives

What are some reading strategies you'll teach in the first two months of school?

# Strategies – Unfamiliar Words

- Sound it out
- Context clues
- Apply known patterns to a new situations

# Strategies – Comprehension

- Predicting
- Estimating
- Hypothesizing
- Make a movie in your mind
- Storyboarding (beginning, middle, end)
- Story elements (character, setting, problem, solution)

# Strategies – Comprehension

- Make a movie in your mind



# Reminder

Most reading skills and strategies are also *thinking* skills and strategies.

**Your Job:  
Focus on and Monitor For  
Sense-Making Above All Else**

# Margaret and Emily's Story



As young teachers, we believed our job was to carefully explain what we knew about mathematics to our students. We asked questions and listened to our students' answers but our listening was aimed at assessing whether our students got what we had explained rather than uncovering their understanding of the content.

We now see that we missed valuable opportunities to develop students' understanding because we did not elicit their ideas or relate their ideas to the content we were teaching.

—Susan B. Empson and Linda Levi  
*Extending Children's Mathematics: Fractions and Decimals*

# Encouraging Sense Making

Q: What's one way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question. Literally.

# Get Rid of the Question

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

**I Notice**

**I Wonder**

# Get Rid of the Question

## ▶ Relate Pictures to Tens and Ones MATH TALK

**MP.1 Make Sense of Problems Analyze the Problem** Discuss the pictures in Exercises 1 and 2. Count the number of cars in the first row. **10 cars** Explain that drivers may be directed to fill a row before parking in the next row of a parking lot. In the same way, people may be asked to fill a row of seats before sitting in the next row at a theater.

- How can a filled row help you count the number of cars or the number of people? **Possible response: A filled row shows ten, so I can use the picture to count tens and extras.**
- How do the cars in Exercise 1 show tens and ones? **2 filled rows show tens and 3 extra cars show ones.**
- How do the people in Exercise 2 show tens and ones? **There are 4 rows of ten with 6 extra ones. This time the ones are at the top and the tens are shown below.**



4-18  
Class Activity

Name \_\_\_\_\_



### ▶ Math and the Community Theater

Linda and her family go to a show.



1. 10 cars can park in each row.



How many cars are there?

\_\_\_\_\_ tens \_\_\_\_\_ ones = \_\_\_\_\_ cars



2. 10 people can sit in each row.



How many people are there?

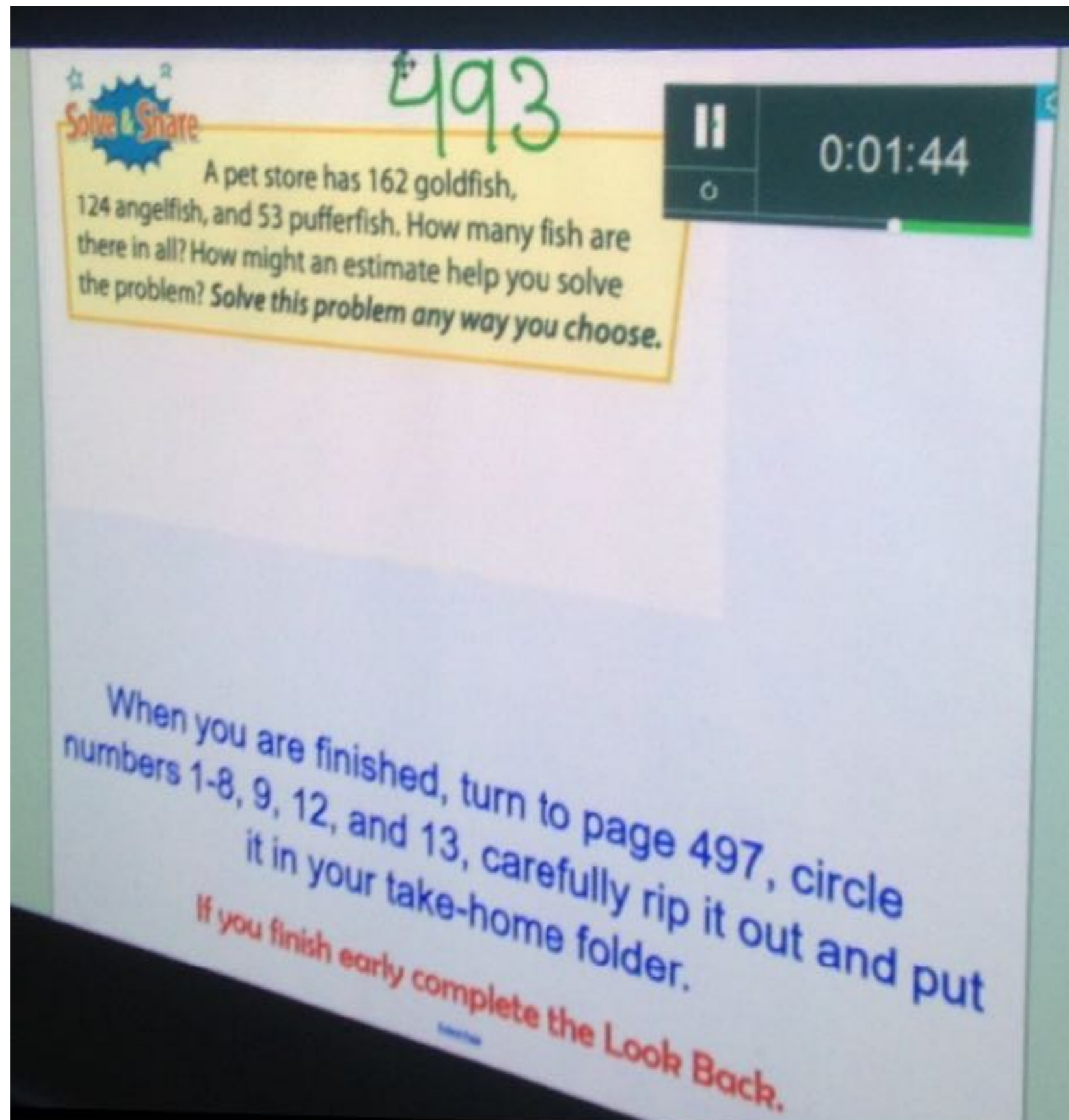
\_\_\_\_\_ tens \_\_\_\_\_ ones = \_\_\_\_\_ people

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# Get Rid of the Question



# Ask for Questions, Not Answers



**Solve & Share**

493

A pet store has 162 goldfish, 124 angelfish, and 53 pufferfish. How many fish are there in all? How might an estimate help you solve the problem? *Solve this problem any way you choose.*

0:01:44

When you are finished, turn to page 497, circle numbers 1-8, 9, 12, and 13, carefully rip it out and put it in your take-home folder.

*If you finish early complete the Look Back.*

# Ask for Questions, Not Answers

Thursday

A pet store has 162 goldfish 124 angelfish, and 53 pufferfish. What questions could **you** come up with?

00:03:39

5 minutes on your own  
3 with a partner  
All together

The image shows a digital whiteboard interface. At the top, there is a toolbar with various icons for editing and navigation. The main content area contains the text of a math problem. The word 'you' is highlighted in red. In the bottom right corner, there is a digital timer showing 00:03:39 and a list of activity options: '5 minutes on your own', '3 with a partner', and 'All together'.

# Ask for Questions, Not Answers

pufferfish. What questions could you come up with?

How much more does the goldfish's has more than the angelfish?

What is the total of all fishes?

How much does the pufferfish and the goldfish have altogether?

How much does the angelfish and



# Ask for Questions, Not Answers

**What could the questions be now?**

how much more is

blah blah blah than blah

blah blah?

# Encouraging Sense-Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question *and* the numbers.

Raul had some pet mice. Xavier gave him some more mice. Now Raul has 8 mice.

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# Get Rid of the Question and the Numbers

Caitlyn is still trying to make brownies for the class. She has the mix and milk but needs to go get eggs. A carton of eggs weighs some **24** pounds. Each carton has **12** eggs. Each carton costs a **1** dollar amount.

How much does one egg weigh?  
(in ounces)

# Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Give the answer and let the students do the work.

# Give the Answer

## ◆ Math Message Follow-Up

### WHOLE-CLASS ACTIVITY

Draw or display a function machine and “What’s My Rule?” table. (See Advance Preparation.)

Ask children to imagine that the **function machine** works like this:

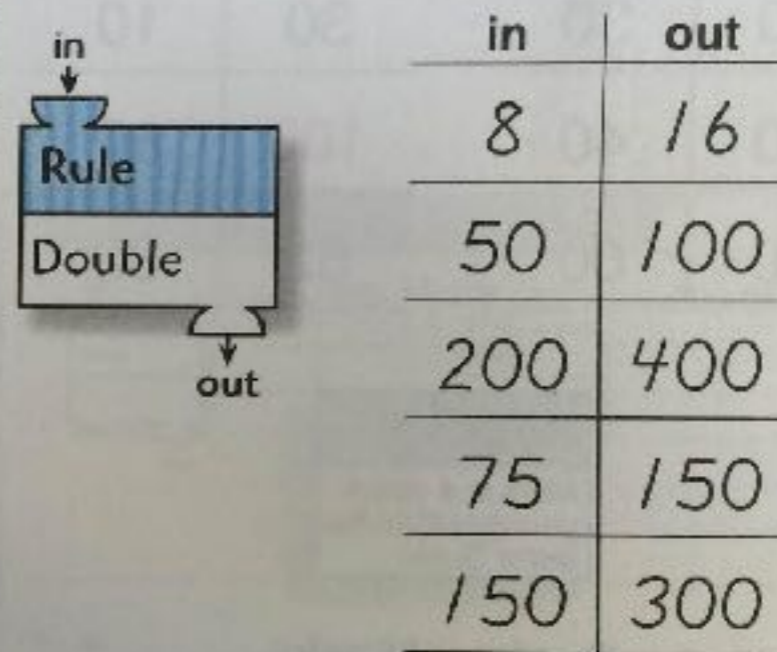
- A number (the **input**) is dropped into the machine,
- the machine changes the number according to a rule,
- and a new number (the **output**) comes out the other end.

The **rule** for the Math Message problem is “Double the number.” Write the word *Double* in the function machine.

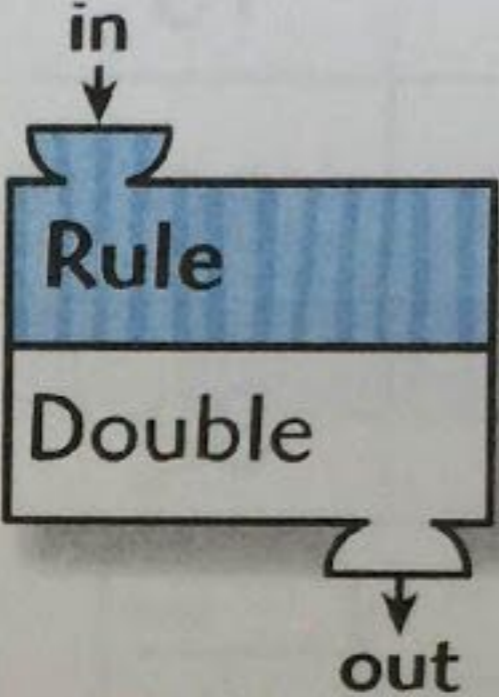
Point out the “**What’s My Rule?**” table. Discuss the 8 in the *in* column and the 16 in the *out* column. Explain to children that numbers in the *in* column represent the numbers of bacteria now. Corresponding numbers in the *out* column represent the numbers of bacteria 20 minutes from now.

Review the answers to the Math Message problem by posing questions in the following manner:

- If 50 is dropped into the **function machine**, which number will come out? **100** Enter the appropriate numbers in the *in* and *out* columns.



# Give the Answer



The diagram shows a function block with an input arrow labeled 'in' and an output arrow labeled 'out'. The block is divided into two sections: 'Rule' (shaded blue) and 'Double'.

in	out
8	16
50	100
200	400
75	150
150	300

# Give the Answer (or Several!)

Rachel bakes cookies and delivers them to her friends.

- It takes 8 minutes to mix the batter.
- The cookies bake for 9 minutes.
- For 6 minutes they cool.

**If the answer is 23 minutes, what is the question?**

**If the answer is 3 minutes, what is the question?**

**If the answer is bake, what is the question?**



# Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Ask about ideas, not answers.

This can be really simple:  
“Tell me something about number 7.”  
*instead of*  
“What’s the answer to number 7?”

# Teacher Questions

“Why?”

“How do you know?”

“How did you decide?”

“Tell me more about that.”

# Ways to Encourage Sense Making Rather Than Answer Getting

- Get rid of the question.
- Get rid of the question *and* the numbers.
- Give the answer.
- Ask about ideas, not answers.

**Your Job:  
Focus on and Monitor For  
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# Reflections?

## We notice ...

- Kindergarteners point and verbally count
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