

# Using Technology to Increase Conceptual Understanding in Algebra and Geometry

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A PDF of the slides will be available after the session at  
[annie.mathematicalthinking.org](http://annie.mathematicalthinking.org)

Links to all the technology will eventually be available as well.



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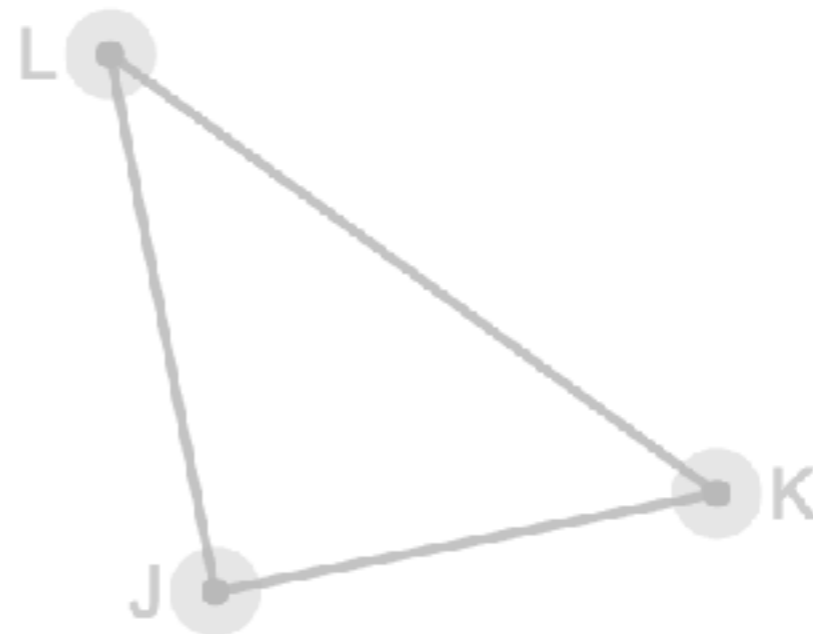
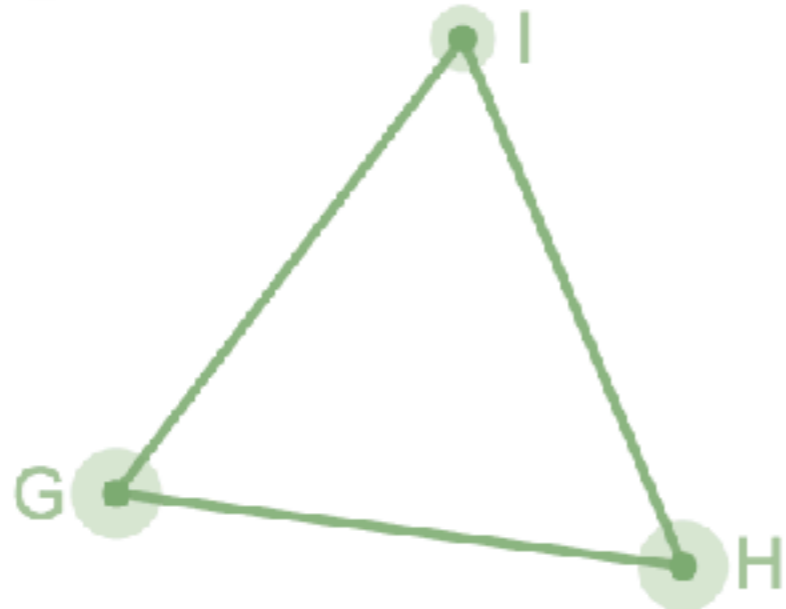
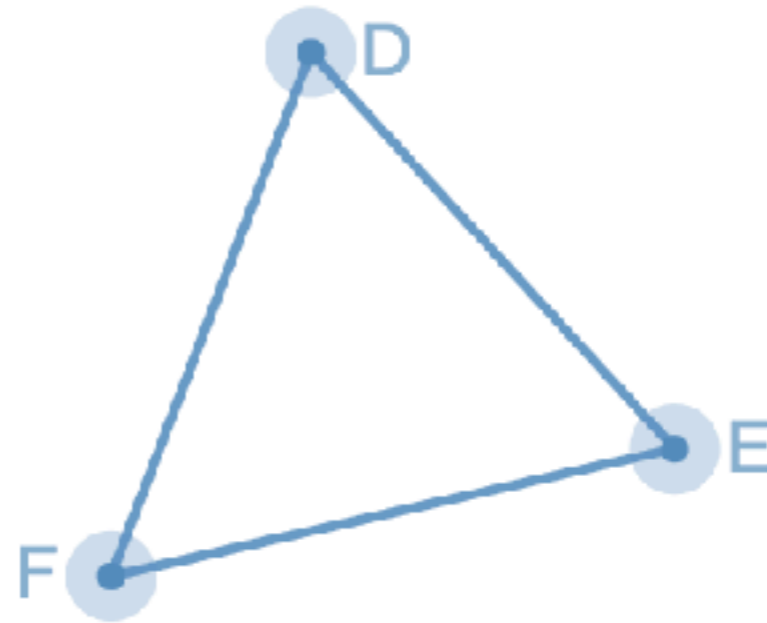
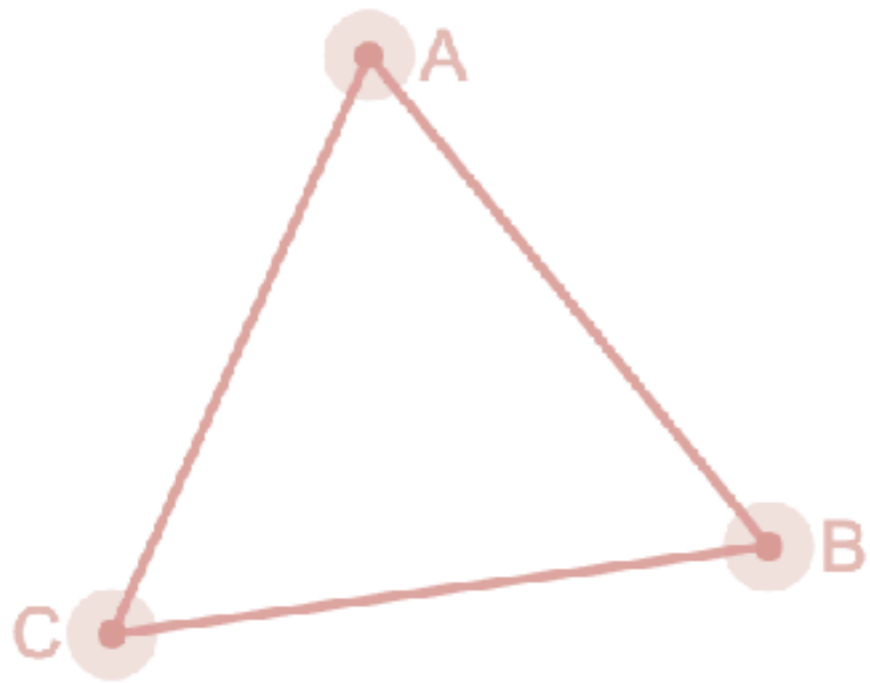
**October 27, 2018**

**Saint Peter's University – Mac Mahon Student Center  
47 Glenwood Avenue, Jersey City, NJ 07306**

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#NoticeWonder

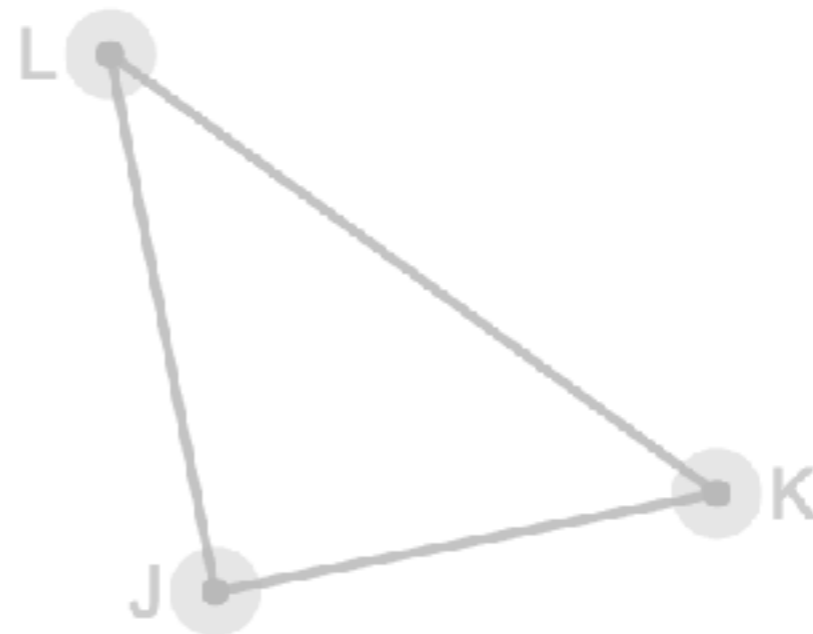
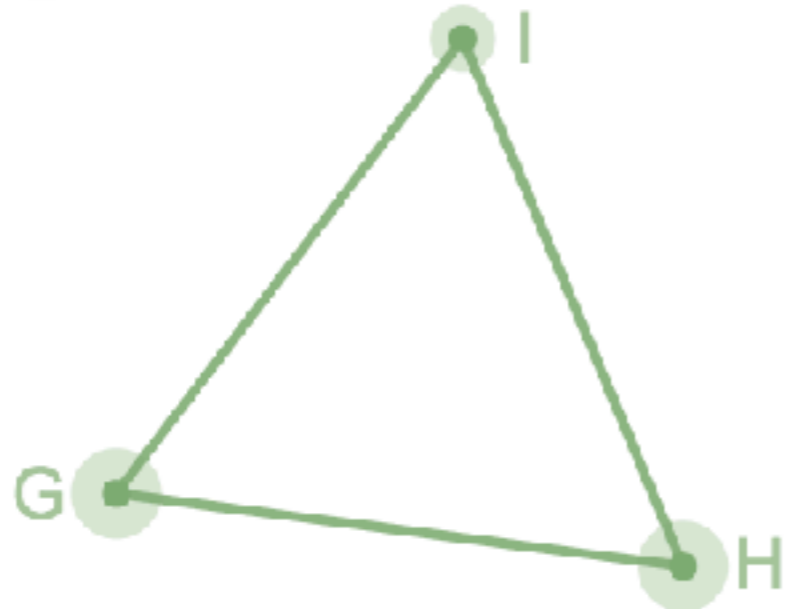
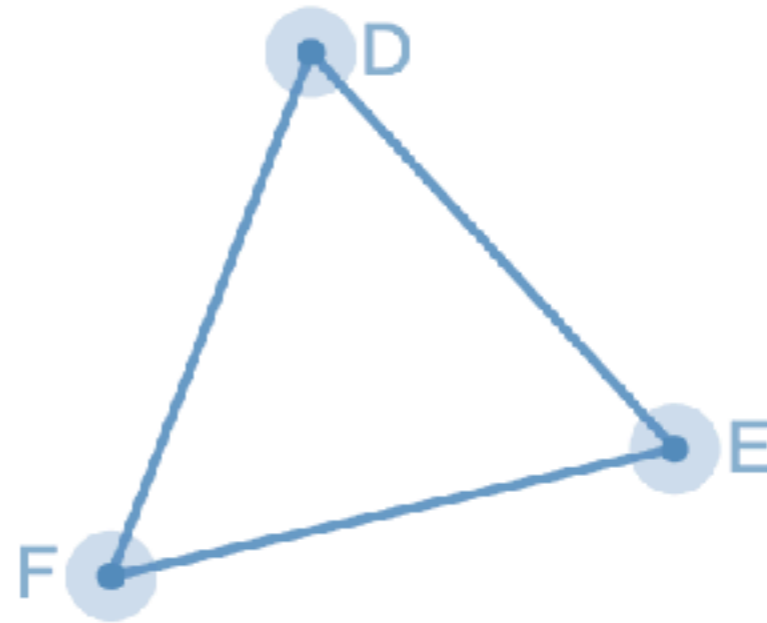
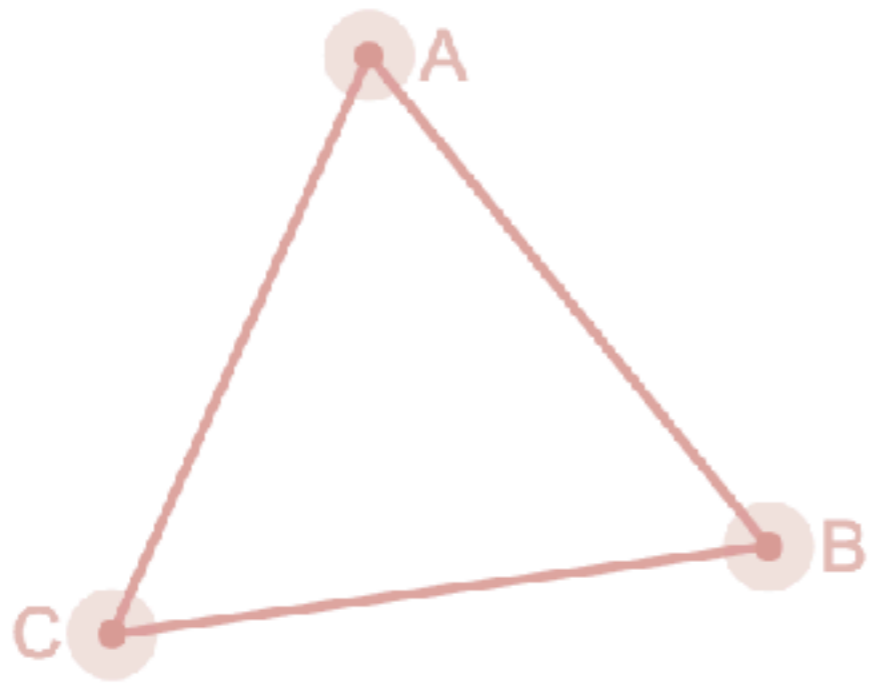
# Types of Triangles

student.desmos.com - Student Code **K7A Z2A**



# Types of Triangles

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# Types of Triangles

	Always True	Sometimes True	Never True
Red			
Blue			
Green			
Gray			

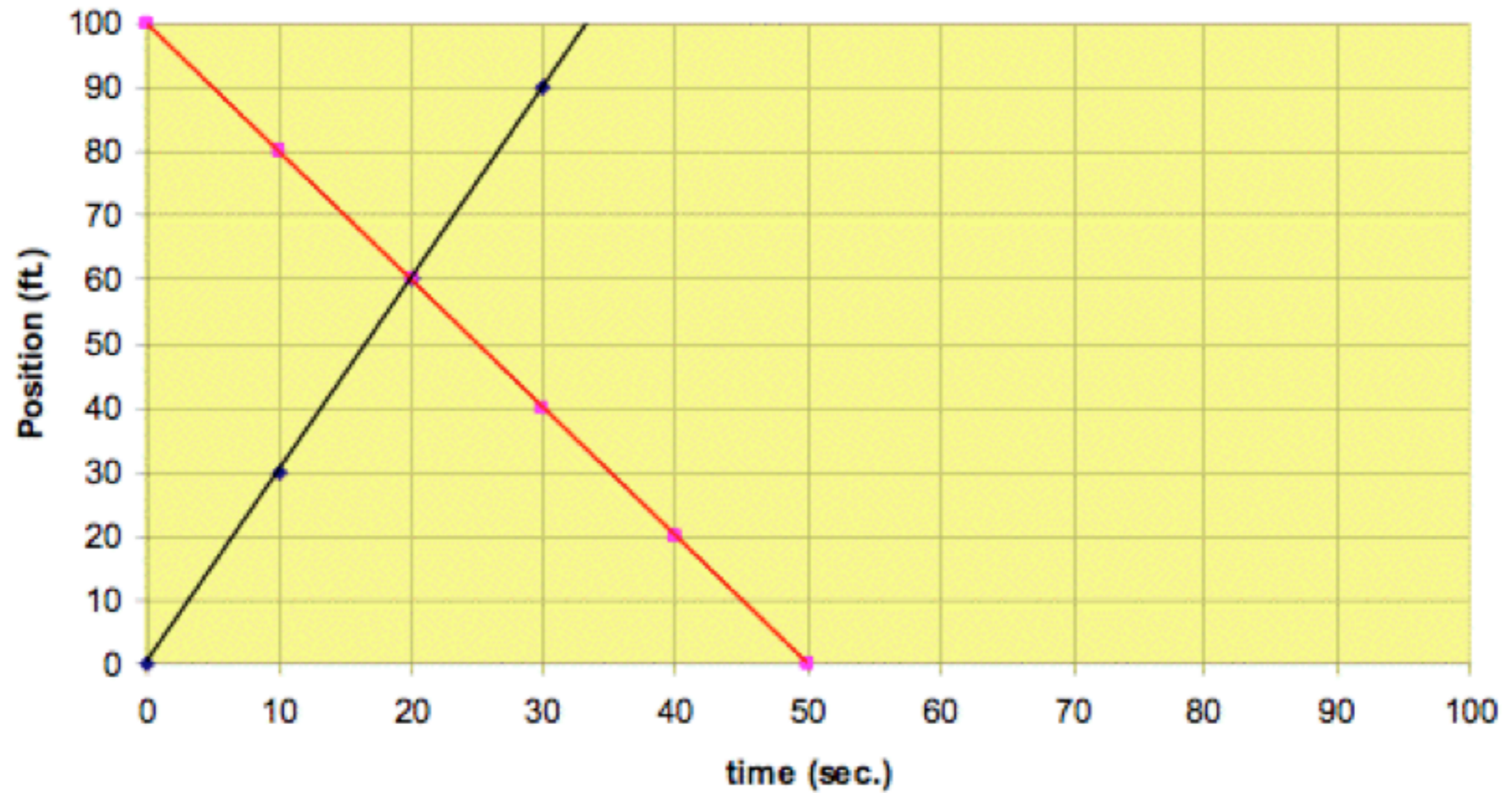
# Types of Triangles

	Red	Blue	Green	Gray
Gray		<b>Y</b>		
Green				
Blue				

# Types of Triangles—Strengths?

- Infinitely many examples of each type.
- Develop intuition about types of triangles.
- No need for formal vocabulary.
- No laundry lists of properties to remember.
- Great formative assessment opportunity.

## Runners



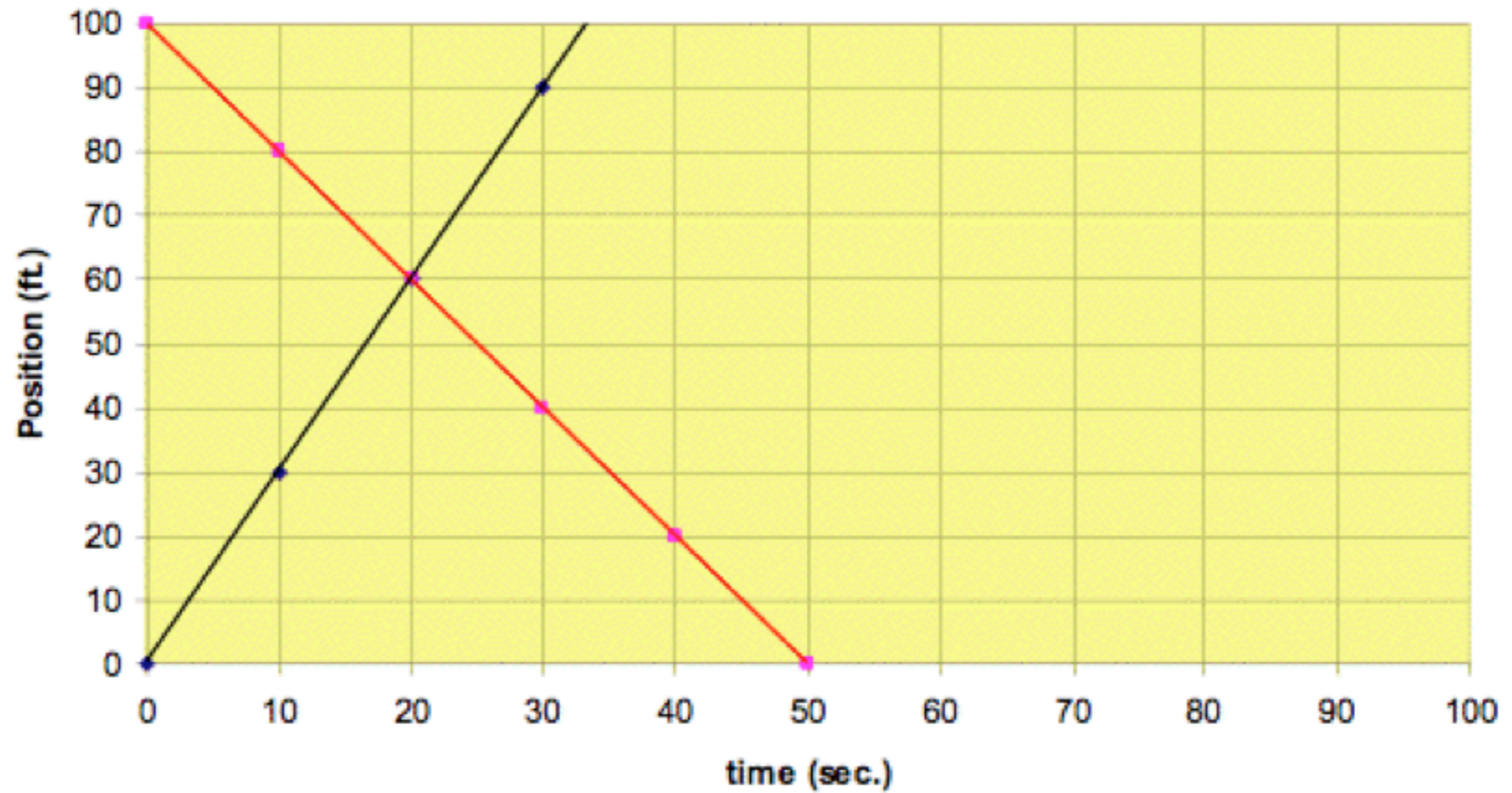


# Runners

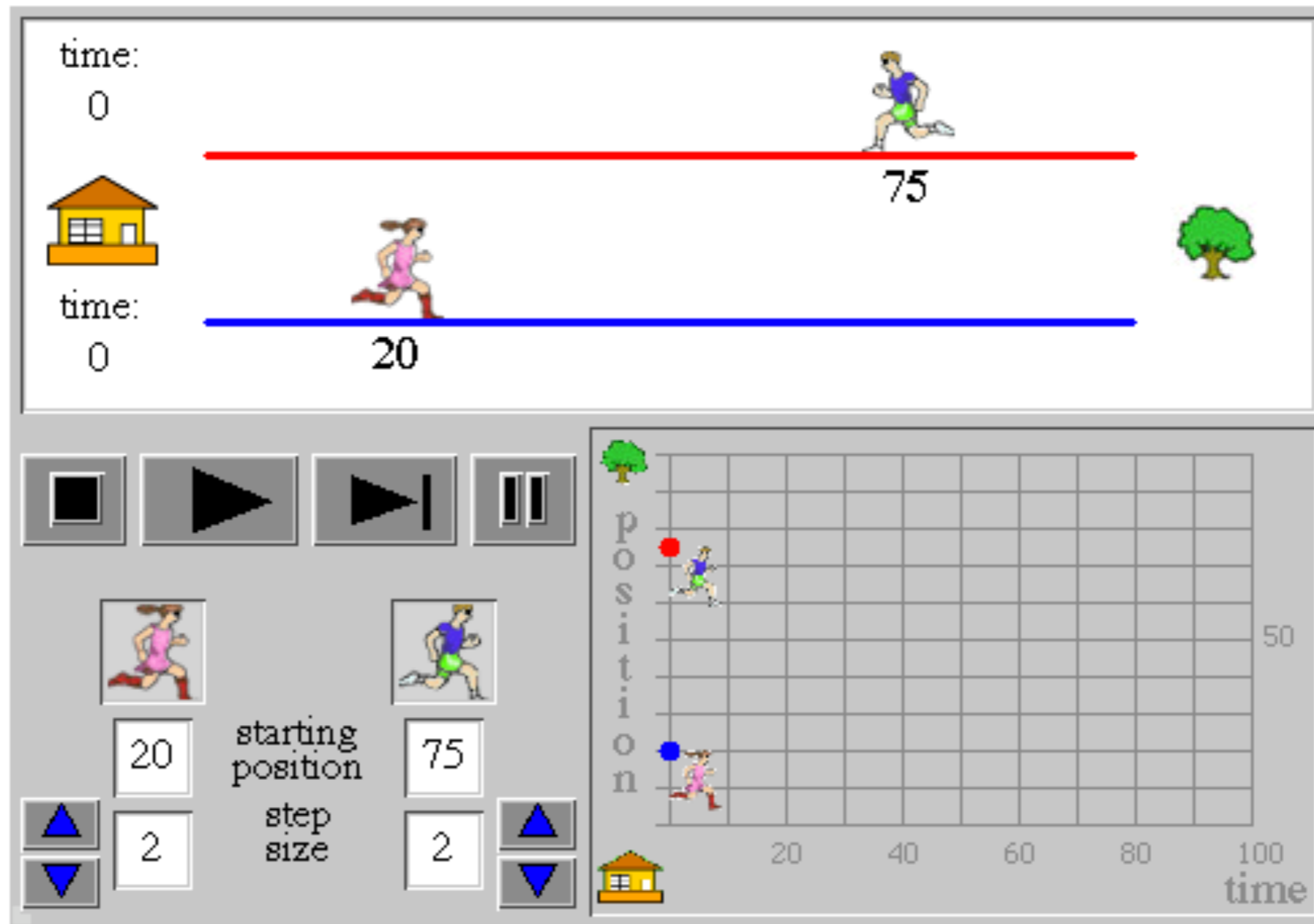
I Notice

I Wonder

## Runners



# Runners



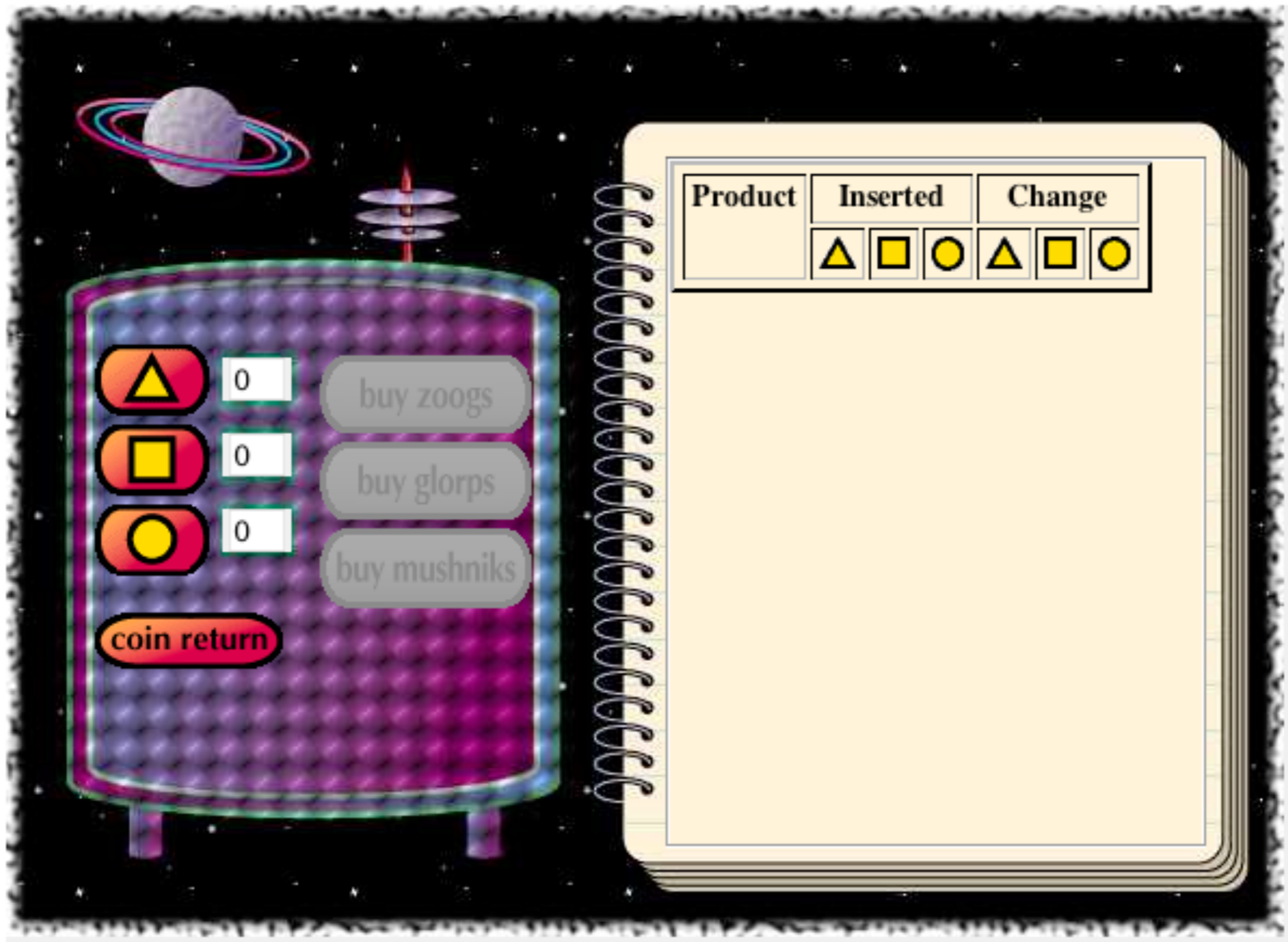
# Runners–Possible Tasks

- Generate the situation shown in the diagram.
- Have the runners start and end at the same time.
- Have the runners generate paths that are parallel.
- Have the runners generate paths that don't intersect but aren't parallel.
- Pick a point on the graph: \_\_\_\_\_ Make the runners generate paths that intersect at that point.
- Generate a task to challenge your classmates.

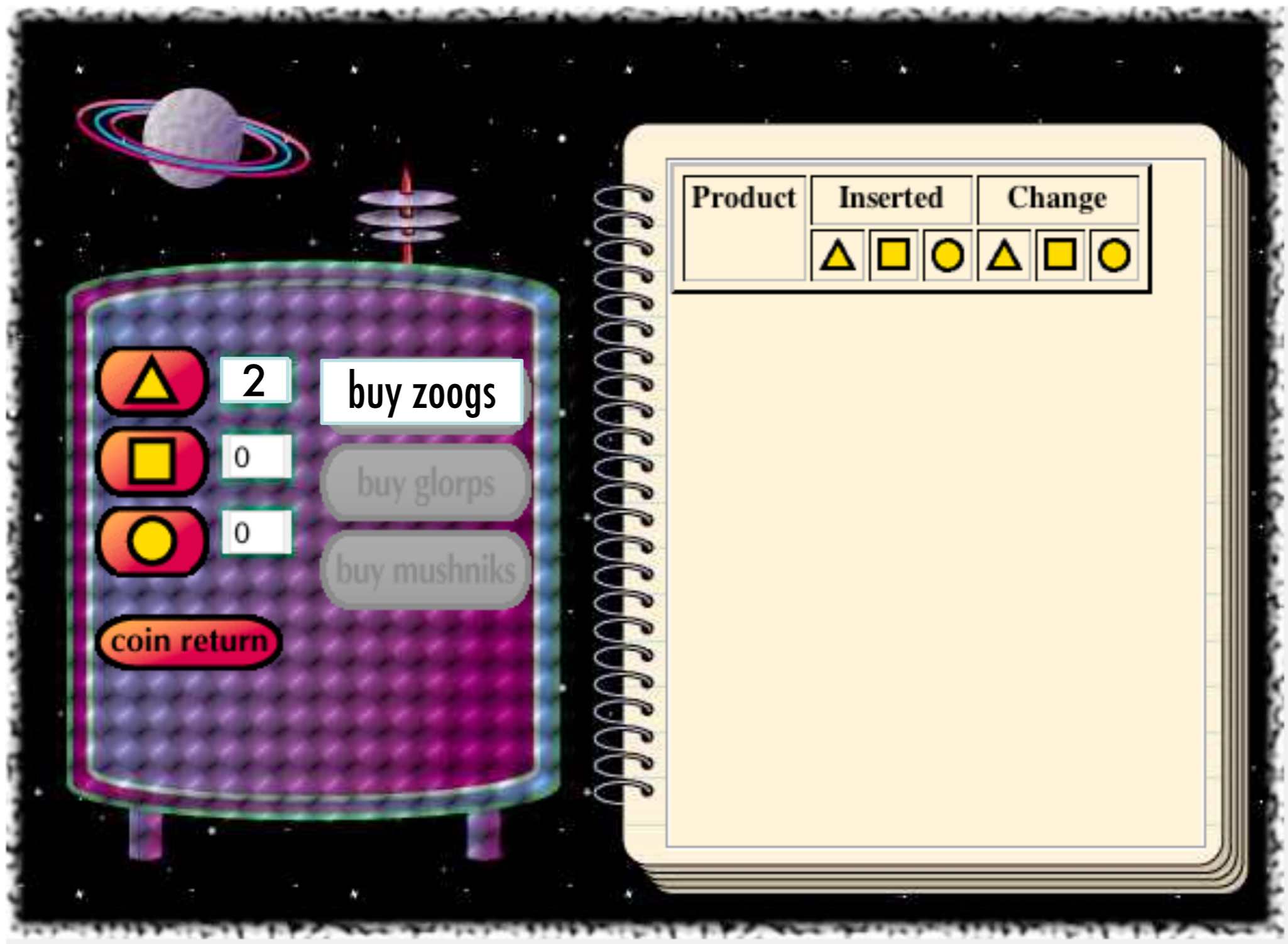
# Runners–Strengths

- Develop intuition about distance, rate, time relationships.
- Develop intuition about slope.
- No need for formal vocabulary.
- Can guess and check as many times as possible since the app gives near-instant feedback.
- Identifying the many points of entry and methods of solving systems of equations, from concept, to methods, to procedures.

# Galactic Exchange



# Galactic Exchange



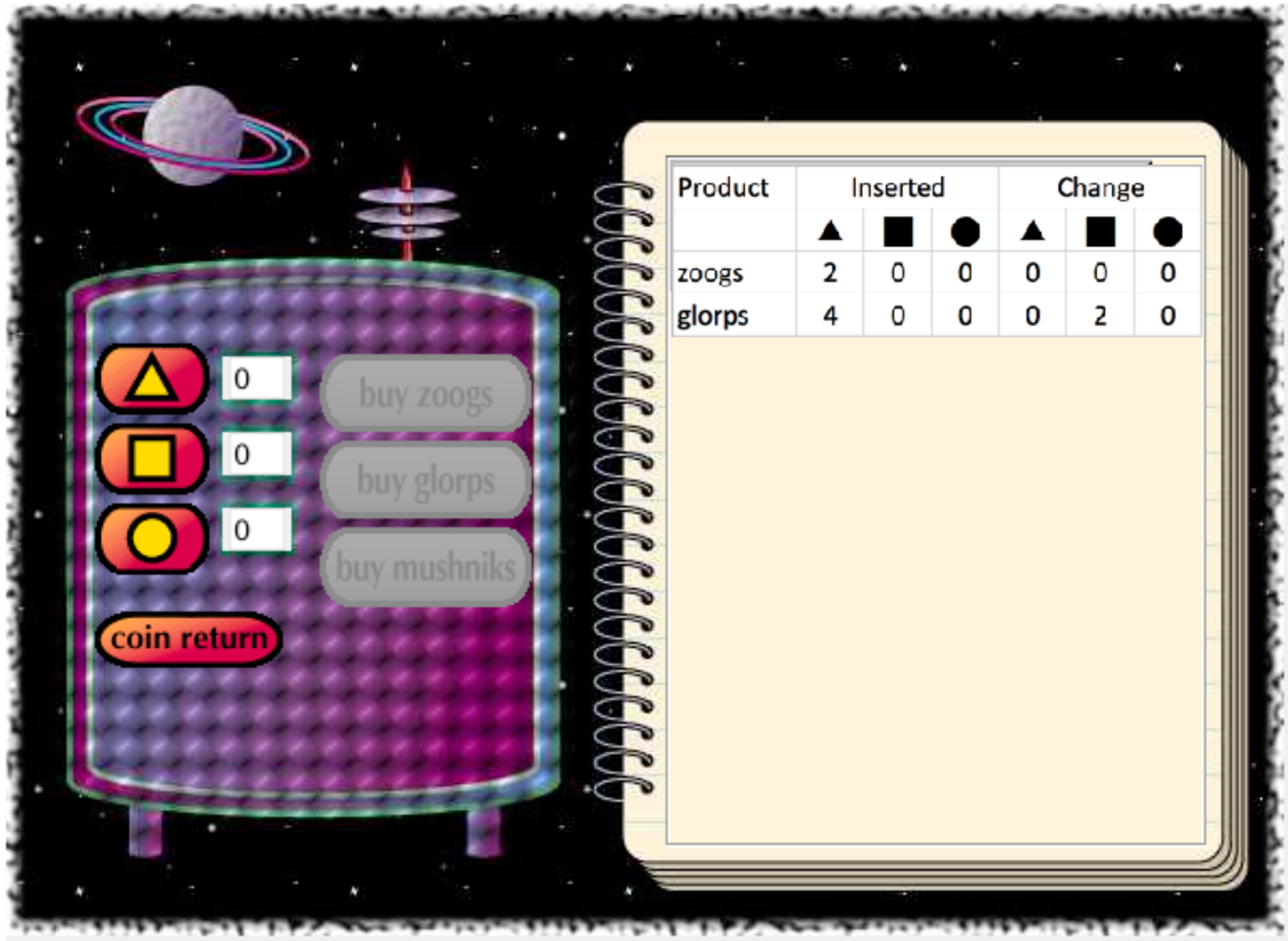
# Galactic Exchange

The image shows a futuristic vending machine on the left and a spiral-bound notebook on the right. The vending machine has three buttons with yellow symbols (triangle, square, circle) and a 'coin return' button. Each button has a digital display showing '0'. To the right of the buttons are three buttons labeled 'buy zoogs', 'buy glorps', and 'buy mushniks'. The notebook has a table with the following data:

Product	Inserted			Change		
	▲	■	●	▲	■	●
zoogs	2	0	0	0	0	0



# Galactic Exchange



The image shows a purple vending machine with three buttons: a yellow triangle, a yellow square, and a yellow circle. Each button has a digital display showing '0'. To the right of the buttons are three buttons labeled 'buy zoogs', 'buy glorps', and 'buy mushniks'. Below these is a 'coin return' button. To the right of the vending machine is a spiral-bound notebook with a table on the first page.

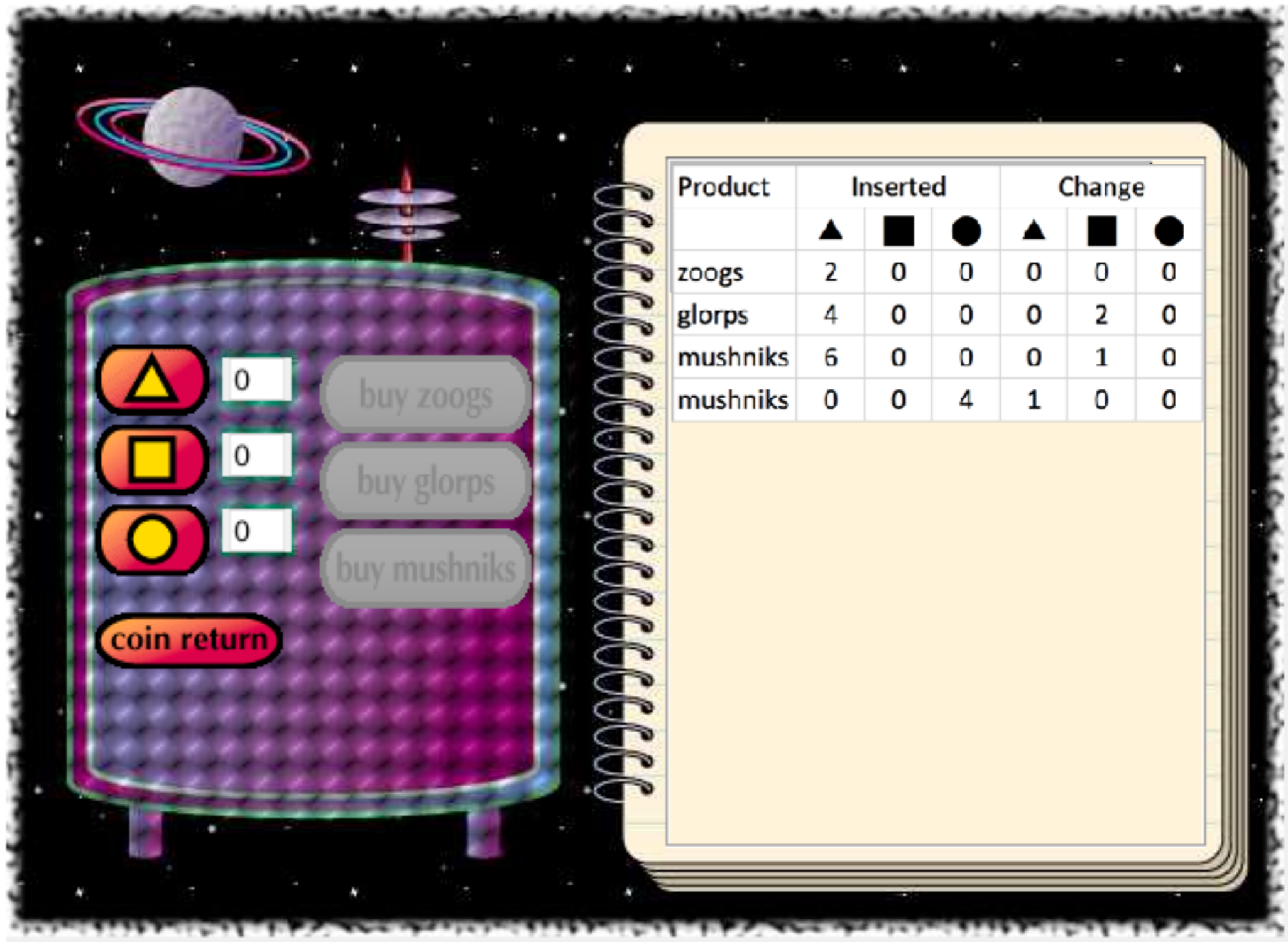
Product	Inserted			Change		
	▲	■	●	▲	■	●
zoogs	2	0	0	0	0	0
glorps	4	0	0	0	2	0

# Galactic Exchange

The image shows a purple vending machine with three coin slots, each containing a yellow coin and a digital display showing '0'. The buttons are labeled 'buy zoogs', 'buy glorps', and 'buy mushniks'. A 'coin return' button is also present. To the right is a spiral notebook with a table of exchange rates.

Product	Inserted			Change		
	▲	■	●	▲	■	●
zoogs	2	0	0	0	0	0
glorps	4	0	0	0	2	0
mushniks	6	0	0	0	1	0

# Galactic Exchange



The image shows a purple vending machine with three buttons: a yellow triangle, a yellow square, and a yellow circle. Each button has a digital display showing '0'. To the right of the buttons are three buttons labeled 'buy zoogs', 'buy glorps', and 'buy mushniks'. Below these is a 'coin return' button. To the right of the vending machine is a spiral-bound notebook with a table on the first page. The table has columns for 'Product', 'Inserted' (with sub-columns for triangle, square, circle), and 'Change' (with sub-columns for triangle, square, circle). The table contains the following data:

Product	Inserted			Change		
	▲	■	●	▲	■	●
zoogs	2	0	0	0	0	0
glorps	4	0	0	0	2	0
mushniks	6	0	0	0	1	0
mushniks	0	0	4	1	0	0

# Galactic Exchange

Use the Galactic Exchange applet to find as many ways as you can to buy something without getting back any change.

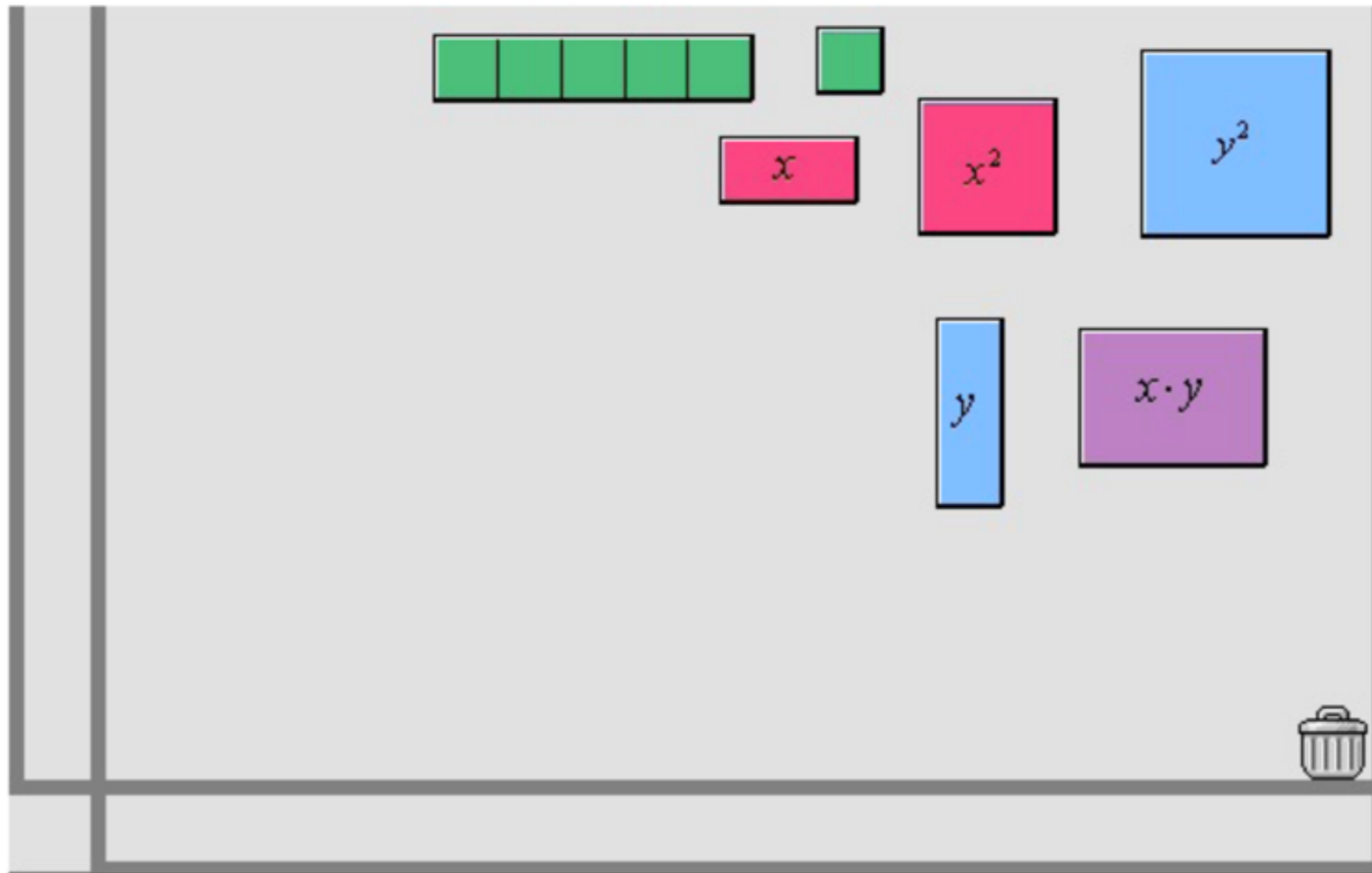
# Galactic Exchange

1. Buy a package of zoogs. What is the fewest number and types of coins needed to purchase a package of zoogs using exact change?
2. Which coin is worth the least and how many does it take to equal each of the other coins?
3. How many of the least valuable coins would it take to buy a package of zoogs?
4. Buy a package of Glorps. What is the fewest number and types of coins needed to purchase a package of Glorps using exact change? How many of the least valuable coins would it take to buy a package of Glorps?
5. Another Earthling arrives on the next shuttle and wants to try some Mushniks. Explain how you figured out the relationship between each coin.
6. What is the fewest number and types of coins needed to purchase a package of Mushniks using exact change?

# Galactic Exchange—Strengths

- user generated data
- record keeping
- identifying the many points of entry and methods of solving systems of equations, from concept, to methods, to procedures

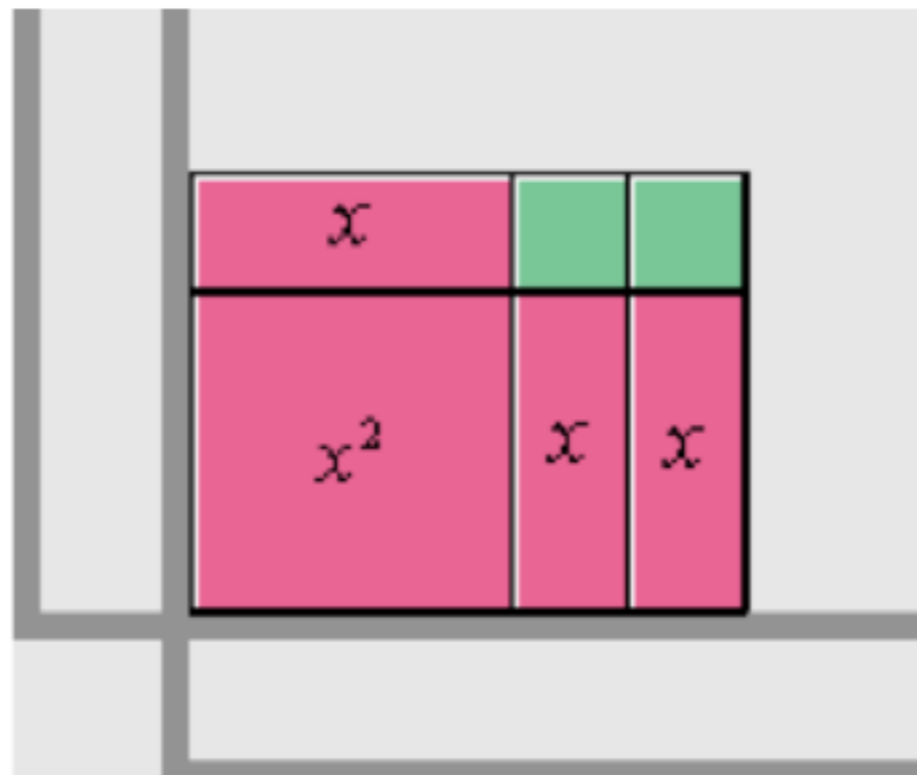
# Algebra Tiles



# Algebra Tiles

## Factoring Using Algebra Tiles

1. Construct the rectangle below using the Algebra Tiles applet:



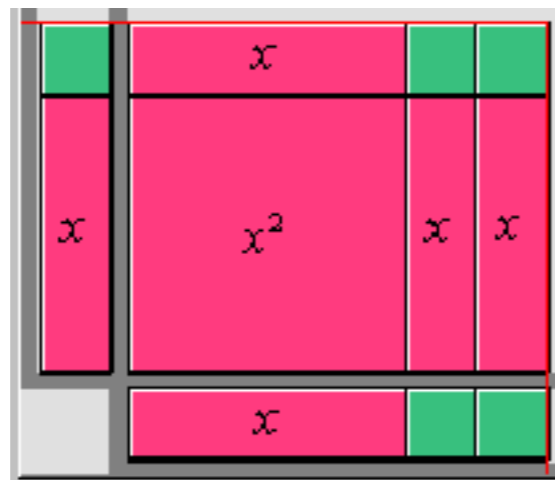
2. Drag the x slider to make sure that the rectangle holds together as the value of x changes.
3. What trinomial is represented by the area of the rectangle?

$$x^2 + x + x + x + 2 = x^2 + 3x + 2$$



# Algebra Tiles

4. Use the tools to build the length and width of this rectangle on the outside. Write the expressions for the length and width below.



$$\begin{aligned} \text{height} &= x + 1 \\ \text{width} &= x + 2 \end{aligned}$$

5. Substitute 3 for  $x$  in the expressions for the length, width, and area of the rectangle. Show the work below. Does the length times the width equal the area?

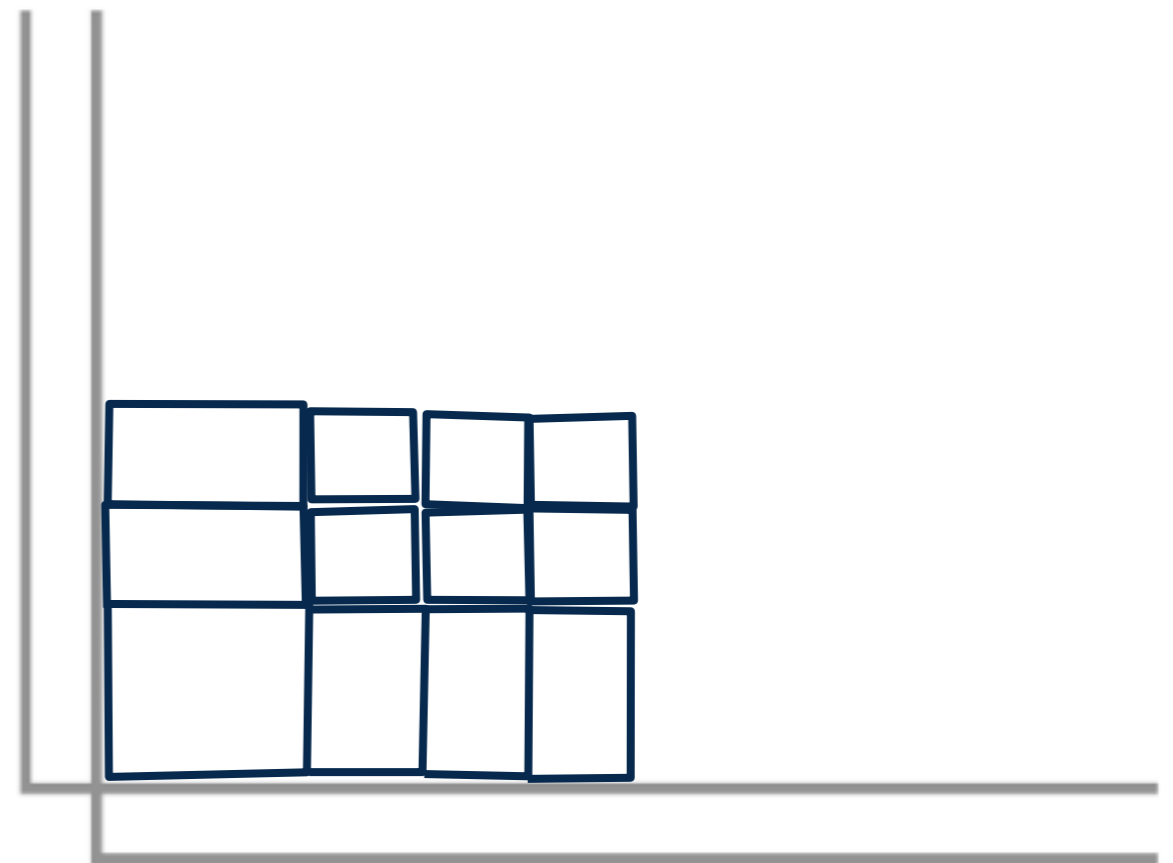
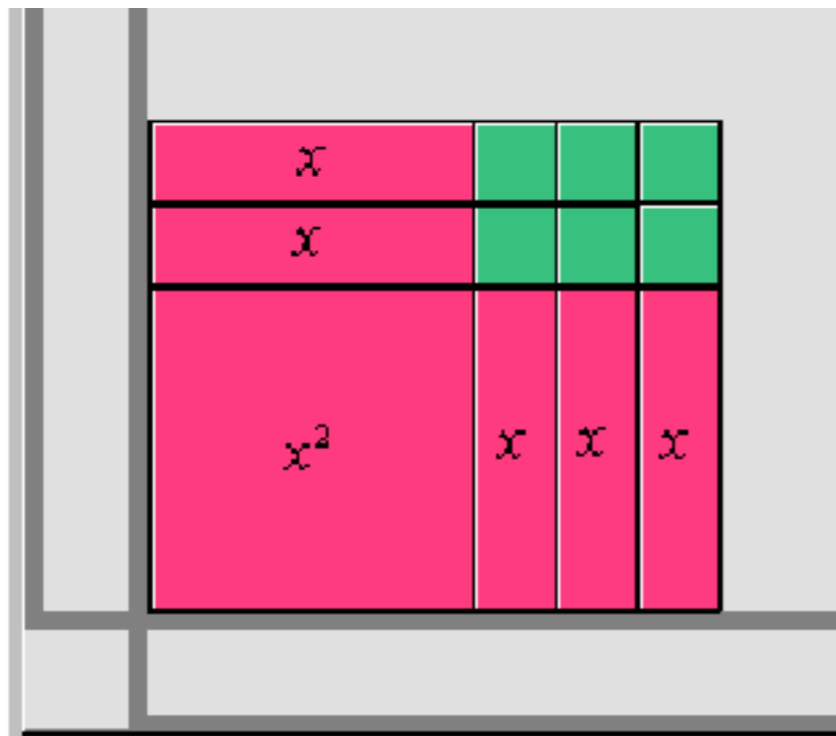
$$\begin{aligned} \text{height} &= x + 1 = 3 + 1 = 4 \\ \text{width} &= x + 2 = 3 + 2 = 5 \end{aligned}$$

$$\text{area} = 4 * 5 = 20$$

$$\begin{aligned} \text{area} &= x^2 + 3x + 2 \\ &= 3^2 + 3(3) + 2 \\ &= 9 + 9 + 2 \\ &= 20 \end{aligned}$$

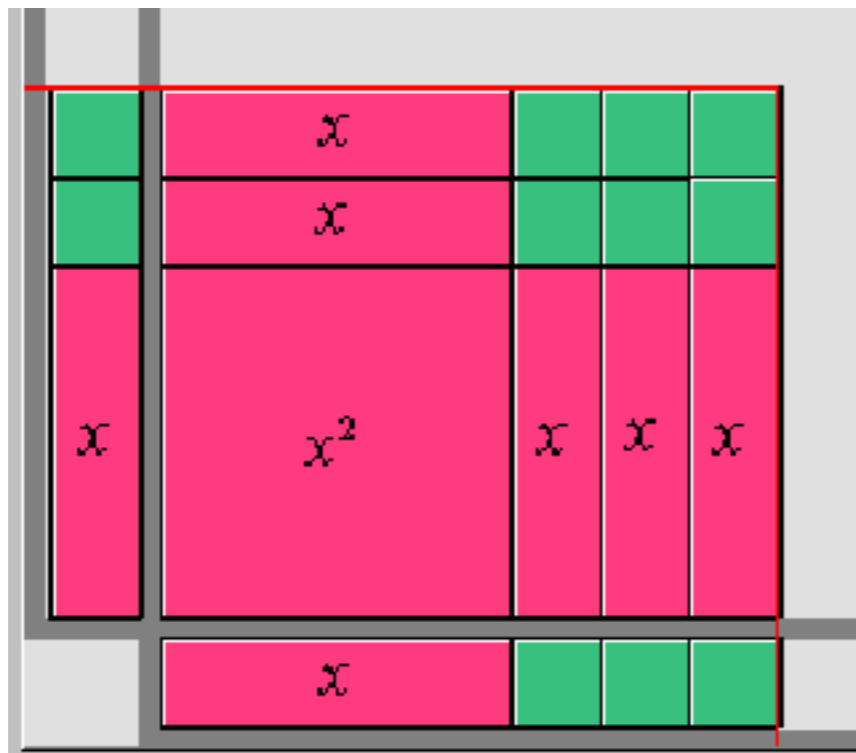
# Algebra Tiles

6. Hit the Clear button in the applet.
7. Use the tools to build a rectangle on the inside to represent the trinomial  $x^2 + 5x + 6$ . Drag the  $x$  slider to make sure that the rectangle holds together as the value of  $x$  changes. Once you have your rectangle, draw it on the first grid on the attached page.



# Algebra Tiles

8. Build the rectangle's length and width on the outside. Write the factored expression represented by these dimensions.



$$(x + 2)(x + 3)$$

# Algebra Tiles

9. Use the applet to build and factor the following expressions. Draw your results on the attached page (please label the grids to show which one goes with which question). Show the factored expressions here.

a.  $x^2 + 7x + 6$

b.  $x^2 + 6x + 8$

c.  $x^2 + 8x + 12$

d.  $2x^2 + 3x + 1$

e.  $4x^2 + 7x + 3$

# Algebra Tiles

10. Try to build and factor the expression  $x^2 + 4x + 6$ . Describe the problem that you encounter.
11. Describe how the second and third terms of a factorable trinomial are related when the leading coefficient (the coefficient of the  $x^2$  term) is 1.

# Principles to Actions

An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.

# What Are You Wondering?



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# Thank You!

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