

Sense-Making: Is It at the Core of Your Math Program?

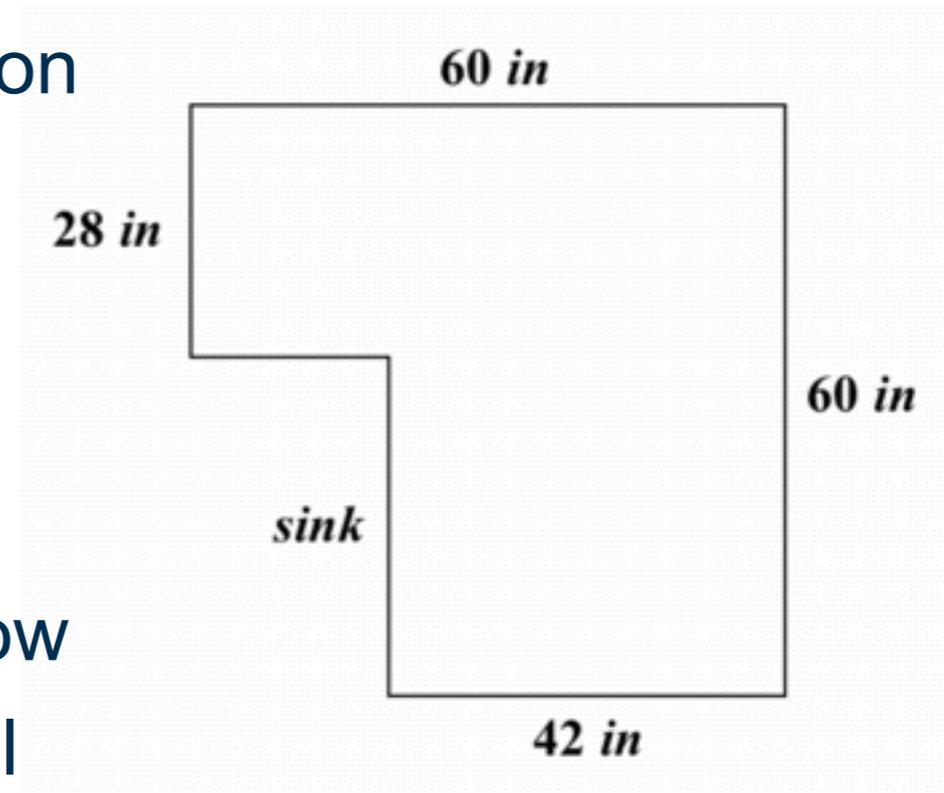
Annie Fetter • anniefetter@gmail.com • [@MFAnnie](https://twitter.com/MFAnnie)

Slides and links to related resources will be available on my blog after the talk:

annie.mathematicalthinking.org

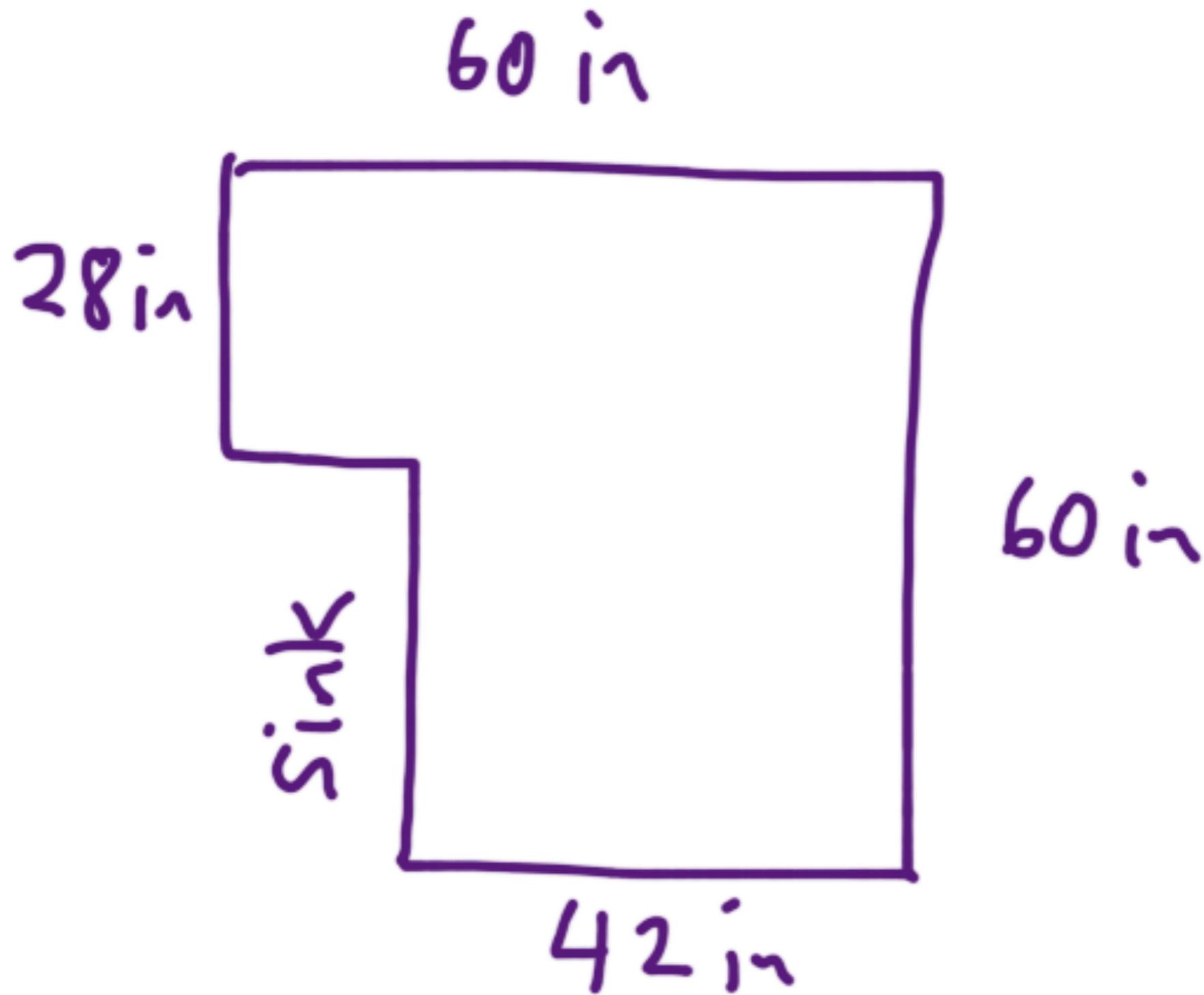
Teresa's Tiles

Teresa is going to put down new ceramic tiles on her bathroom floor. She has selected square tiles that are 4 inches on each side. These are the kind of tiles that can be placed right next to each other without leaving additional space for grout. At The Home Station, she learned how to cut the tiles in case she needs any fractional pieces to cover her floor completely.



This diagram of the bathroom floor shows the dimensions of the floor space she needs to cover. The sink area does not get tiled.

Questions: How many tiles will she need to buy to cover her floor?
How many tiles will she have to cut in order to cover the entire space?



Teresa's Tiles "Scenario"

Things that some "low-performing" 8th graders noticed about the picture:

- two sides are equal
- two sides are 60 inches
- one side is 28 inches
- they are longest
- one side is 42 inches
- it used to be a square
- your lines aren't very straight
- the short side of the sink is 18"
- the sink is a rectangle
- the long side of the sink is 32"
- can find the area of the whole thing by making it two pieces

**There are 125 sheep and
5 dogs in a flock.
How old is the shepherd?**

Robert Kaplinsky (@robertkaplinsky) gave this to 32 eighth grade students. How many do you think attempted to find a numerical answer?

How Old is the Shepherd?

75% gave numerical answers.

100% of his sixth graders gave numerical answers.

In the original research paper [Reusser 1986],
“...three out of four school children will
produce a numerical answer to this problem.”

<http://robertkaplinsky.com/how-old-is-the-shepherd/>
<https://www.youtube.com/watch?v=kibaFBgaPx4>

Sample Grade 3 Test Question

The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?

A. 6 18%

B. 9 46%

C. 18 31%

D. 24 4%

Combined scores of the 160 third graders in a group of four low-performing schools I used to support.

Sample Test Question Revised

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.



CCSS Math Practice 1

Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt.**

They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

They monitor and evaluate their progress and change course if necessary.

Sample Grade 3 Test Question

Hot dog buns come in packages of 8. Michael buys 6 packages of hot dog buns. How many hot dog buns does Michael have in all?

- A. 14 43%
- B. 36 8%
- C. 48 40%
- D. 56 5%

“Cracking the Math Code”

ADDITION

Add
Altogether
And
Both
How many
How much
In all
Increased by
Plus
Sum
Together
Total

SUBTRACTION

are not
change
decreased by
difference
fewer
have left
how many did not
have
how many more
less than
remain
subtract
take away
Taller/shorter

MULTIPLICATION

By (dimension)
Double
Each group
Multiplied by
Of
Product of
Times
Triple

DIVISION

as much
cut up
divided by
each group has
half (or other
fractions)
how many in each
parts
quotient of
Separated
Share something
equally
split

(document from the web site of a large Eastern US metropolitan school district)

Sample Grade 3 Test Question

Hot dog buns come in packages of 8. Michael buys 6 packages of hot dog buns. How many hot dog buns does Michael have **in all**?

- A. 14 43%
- B. 36 8%
- C. 48 40%
- D. 56 5%

Integrated Math 2 (Grade 10)

	Boy	Girl	Total
Wear Sunscreen	84	133	217
Do Not Wear Sunscreen	170	118	288
Total	254	251	505

$P(\text{wears sunscreen})$

$P(\text{is a boy})$

$P(\text{wears sunscreen} \mid \text{is a boy})$

$P(\text{wears sunscreen and is a boy})$

$P(\text{wears sunscreen or is a boy})$

$P(\text{is a boy} \mid \text{wears sunscreen})$

$P(\text{is a boy or a girl})$

“Doing Math” or Sense Making?

$$12 - p = 5$$

$$12 - ? = 5$$

[Michelle's son] was struggling to “remember” $28/4$. When [she] asked him, “How do you think about $28/4$?” He replied, “Mom, you aren’t supposed to think about it, you are just supposed to do it!!”

Teacher Knows Best

$$\frac{3}{4} + \frac{5}{8} = \frac{8}{12}$$

Student Perceptions of Math and Sense Making

1. You aren't supposed to sense-make when doing math.
2. You are supposed to use rules and algorithms and accept whatever answer results.
3. You are supposed to do what your teacher said, even when it doesn't seem like a good idea.

The Five Strands of Mathematical Proficiency

National Research Council, 2001, *Adding it up: Helping children learn mathematics*.

1. Conceptual understanding
2. Procedural fluency
3. Strategic competence
4. Adaptive reasoning
5. Productive disposition

“Productive disposition is the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.”

Your Most Important Job: Monitor for Sense-Making

Characteristics of Strong ~~Readers~~ Mathematicians

- They are motivated to ~~read.~~ tackle problems
- They are able to ~~read words~~ accurately and automatically. recite facts
- They comprehend what they read.
- They are able to read with expression.
- They use a variety of strategies to tackle ~~words~~ they don't recognize. problems
- They use active problem solving strategies to search for information, to determine meaning, to make sense of words, to make connections.

Encouraging Sense Making

Q: What's one way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question. Literally.

Get Rid of the Question

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

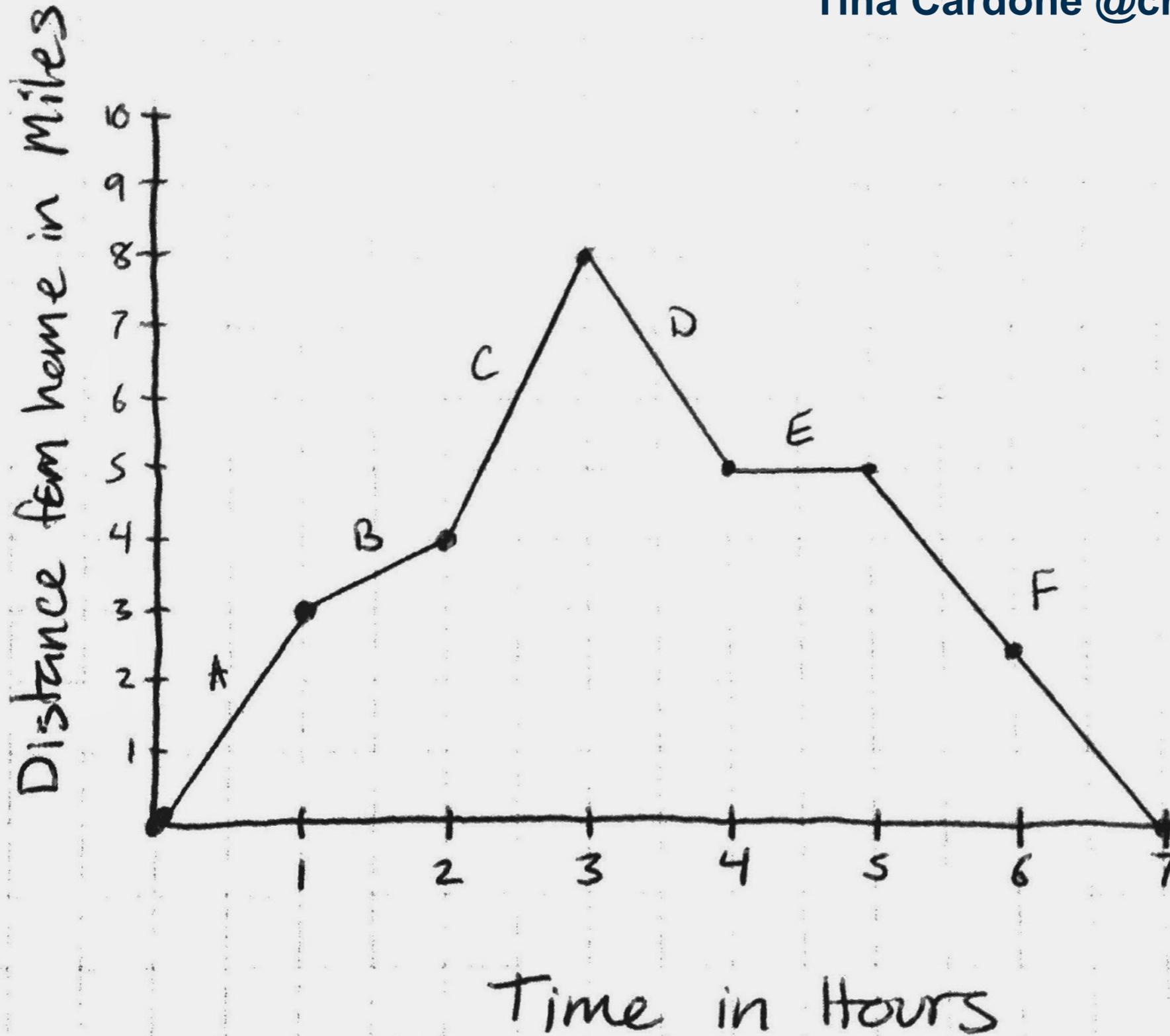
I Notice

I Wonder

Get Rid of the Question

Mr. Gavin has a ladder that is 100 centimeters tall.

Ms. Cornell has a ladder that is 2 meters tall.





Tina Cardone @crstn85 · Nov 24

@MFAnnie when I gave the graph and did notice/wonder first I didn't have to answer nearly so many questions when they did the handout



Tina Cardone @crstn85 · Nov 24

@MFAnnie worth the few minutes it took and meant we skipped wrap up discussion (they already had it)
drawingonmath.blogspot.com/2014/11/distan...



<http://drawingonmath.blogspot.com/2014/11/distance-graph.html>

Encouraging Sense-Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question *and* the numbers.

Get Rid of the Question and the Numbers

Raul had some pet mice. Xavier gave him some more mice.

Raul had some pet mice. Xavier gave him 3 more mice.

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice.

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice. How many mice did Raul have to start with?

A Numberless Word Problem from Brian Bushart, bstockus.wordpress.com

Give the Answer

Caitlyn is still trying to make brownies for the class. She has the mix and milk but needs to go get eggs. A carton of eggs weighs some **24** pounds. Each carton has **12** eggs. Each carton costs a **1** dollar amount.

How much does one egg weigh?
(in ounces)

6

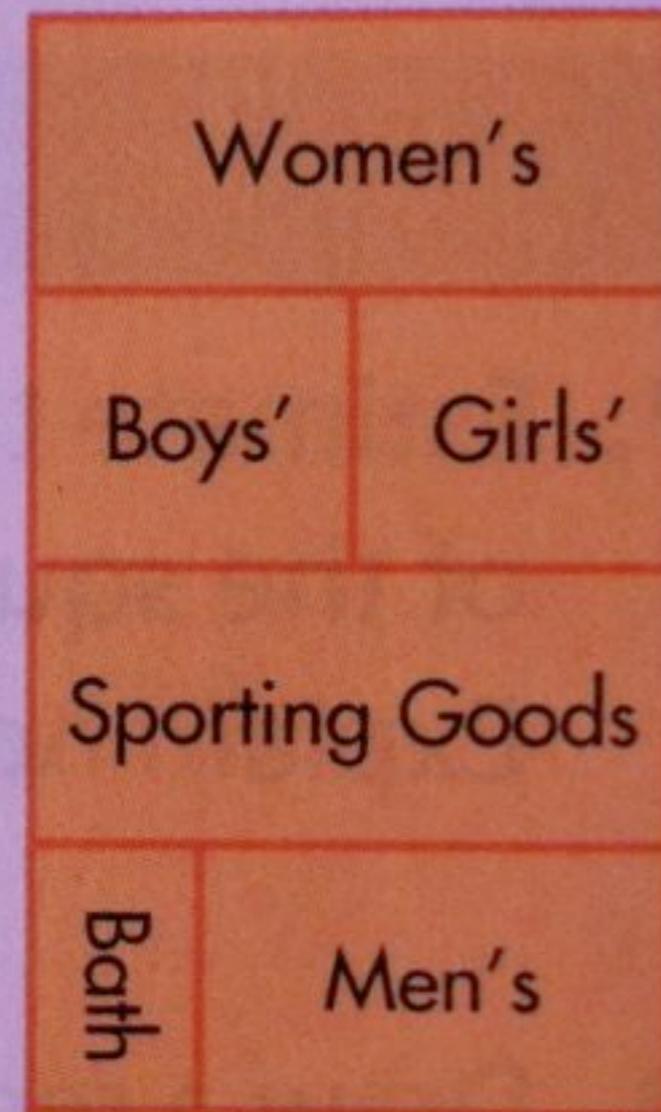
Caitlyn realized that she needs one more ingredient. She forgot the vegetable oil. The oil is sold in **2** liter bottles. She needs a certain number of **6** bottles for the brownies. Each bottle cost a **6.25** dollar amount. Caitlyn brings \$20 with her to the store.

\$5 **1p** **4c**

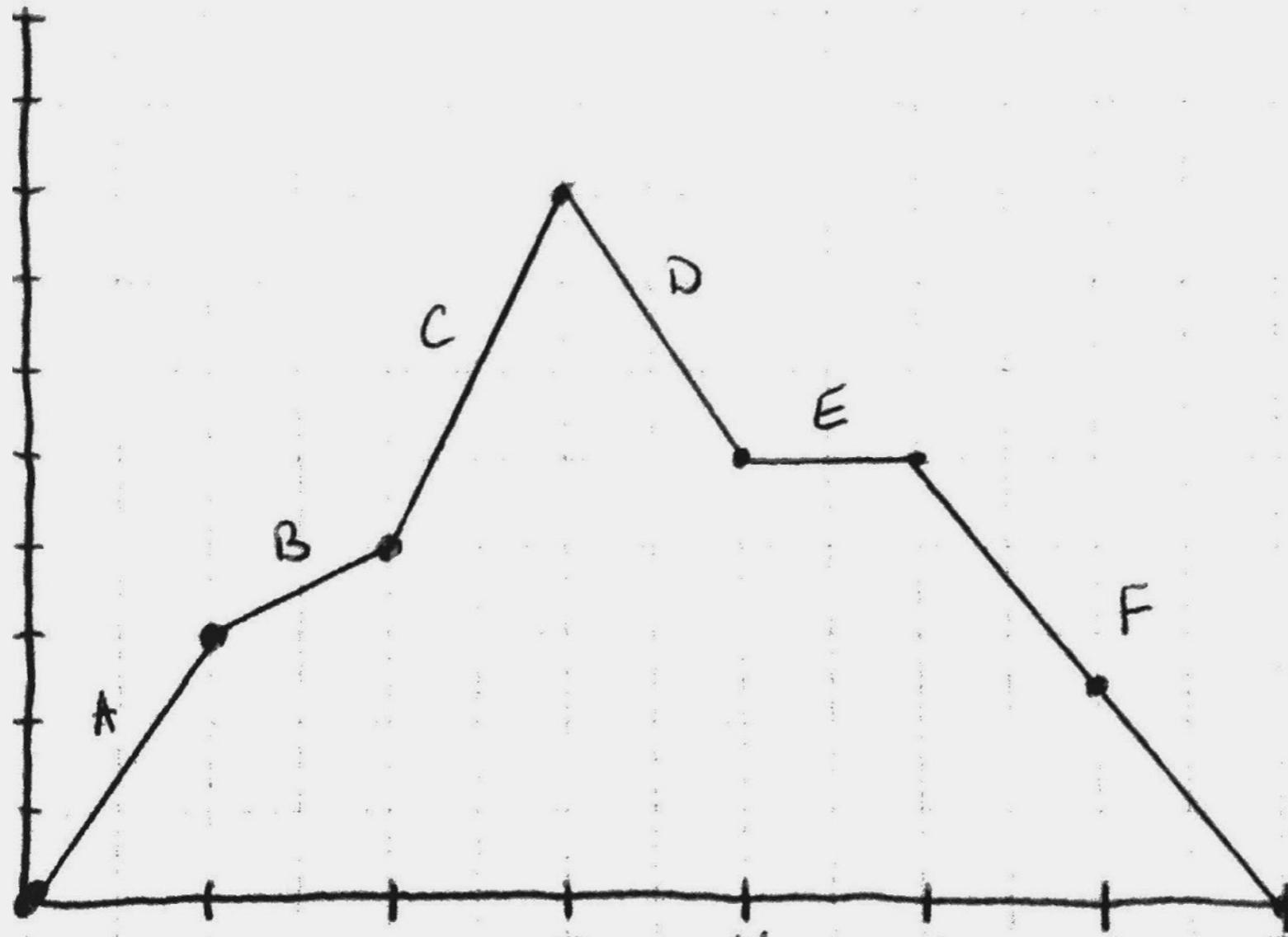
From Kat Kulis, grade 5, Windham Center School, Windham, CT

Get Rid of the Question and the Numbers

A store has the floor plan shown. The area of the women's department is



Distance from home in Miles



Time in Hours

Numbers vs. Relationships

$$A = L \times W$$

$$A = \pi r^2$$

Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Give the answer.

Give the Answer

◆ Math Message Follow-Up

WHOLE-CLASS ACTIVITY

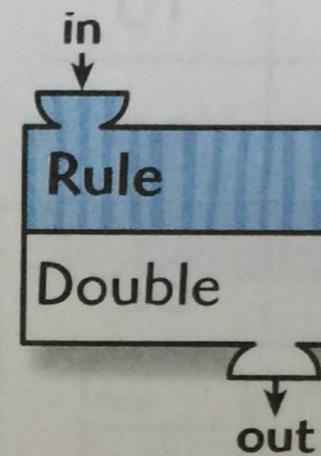
Draw or display a function machine and “What’s My Rule?” table. (See Advance Preparation.)

Ask children to imagine that the **function machine** works like this:

- A number (the **input**) is dropped into the machine,
- the machine changes the number according to a rule,
- and a new number (the **output**) comes out the other end.

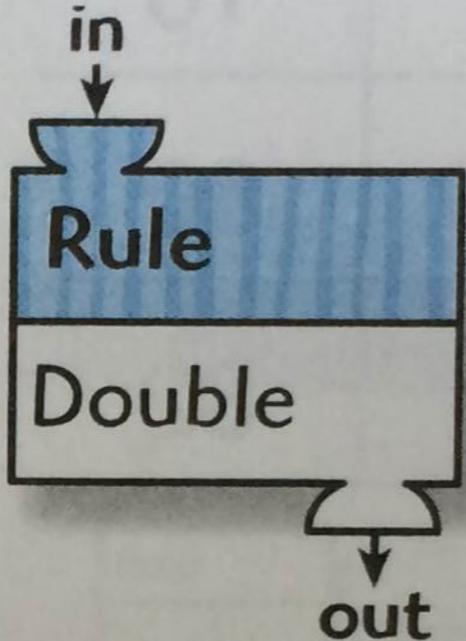
The **rule** for the Math Message problem is “Double the number.” Write the word *Double* in the function machine.

Point out the “**What’s My Rule?**” table. Discuss the 8 in the *in* column and the 16 in the *out* column. Explain to children that numbers in the *in* column represent the numbers of bacteria now. Corresponding numbers in the *out* column represent the numbers of bacteria 20 minutes from now.



in	out
8	16
50	100
200	400
75	150
150	300

Give the Answer



in	out
8	16
50	100
200	400
75	150
150	300

You know $2.4 * 1 = 2.4$. Will $2.4 * 1.8$ be greater than or less than 2.4? How do you know?

Math Message 6.8

"Notice and Wonder" - "I notice....."

$2.4 * 1 = 2.4$
 $2.4 * 1.8 = 4.32$

What do you notice?

- When you multiply by 1, you get same answer as what you multiplied by.
- They multiplied $8 * 4 = 32$ → and they multiplied $4 * 4$.
- $2.4 < 4.32$
- Second problem is wrong. → It's not multiplying, it's adding.
- Both problems have 2.4
- ~~Take out~~ the 3 for swapping 2 and 4 and get same answer as problem 1.
- 4.32 is nearly 2 times as much 2.4
- Can't multiply $2 * 1 = 4$.

Math Message 6.8

"Notice and Wonder" - "I notice....."

$2.4\frac{1}{3} * 1 = 2.4\frac{1}{3}$
 $2.4 * 1.8 = 4.32$

What do you notice?

- It's the same decimals. → 2.4 is the same as answer.
- When multiplying, the product is less.
- The first problem equals the same number.
- By adding $\frac{8}{10}$ to 1, the answer was changed to 4.32.
- Almost all the same numbers except $\frac{8}{10}$.
- The decimals in the ones place is adding up to 2.
- In the second problem it looks like the answer is incorrect because the whole #'s and decimals multiplied would give 2 instead of 4.
- Both problems contain (2) two's and (2) 4's.



Sally Nordyke, Grade 5, Daniel F. Ryan School 19, Passaic, NJ

Give the Answer (or Several!)

Rachel bakes cookies and delivers them to her friends.

- It takes 8 minutes to mix the batter.
- The cookies bake for 9 minutes.
- For 6 minutes they cool.

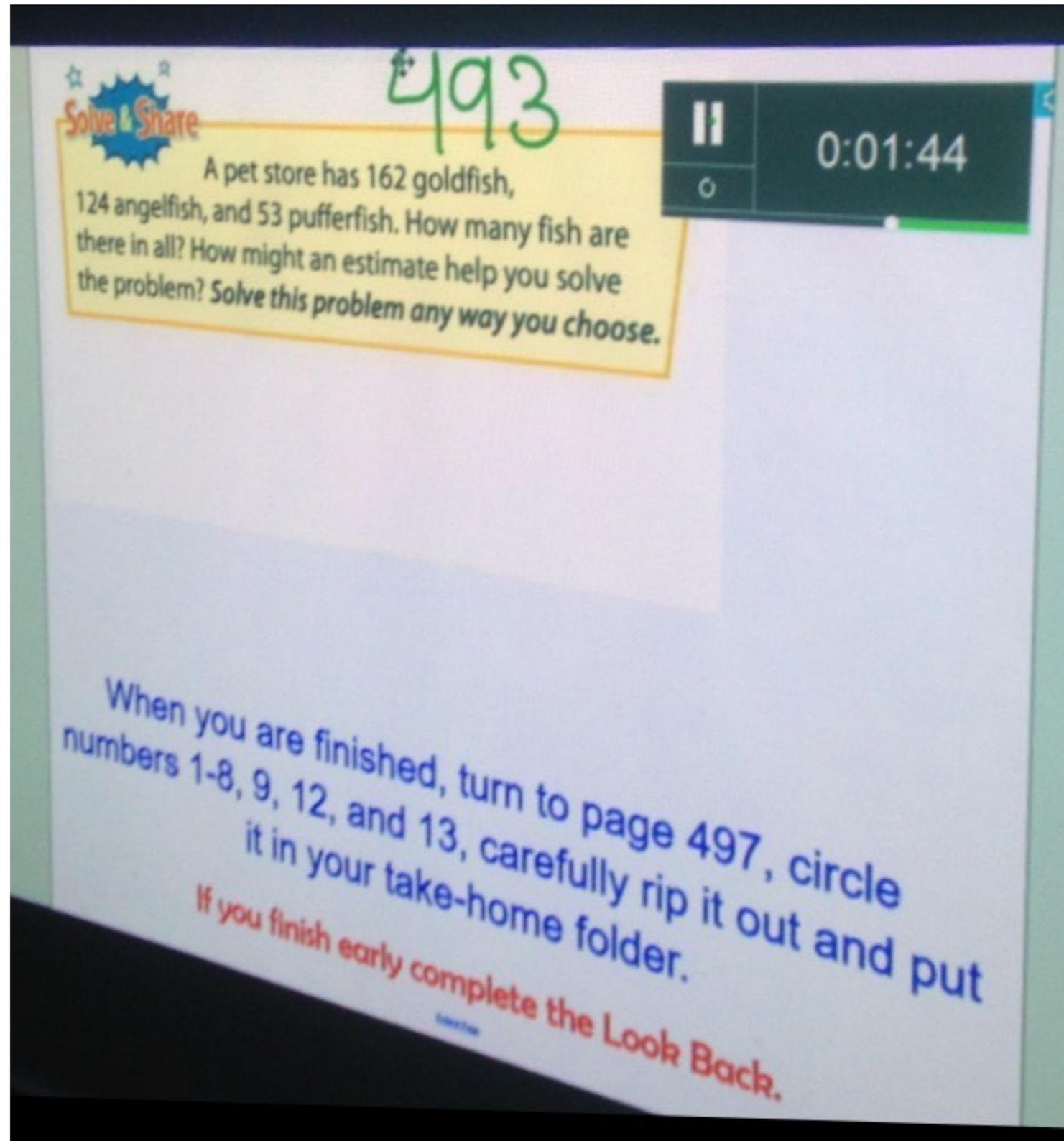
If the answer is 23 minutes, what is the question?

If the answer is 3 minutes, what is the question?

If the answer is bake, what is the question?

from Joe Schwartz, @JSchwartz10a

Ask for Questions



Solve & Share

493

A pet store has 162 goldfish, 124 angelfish, and 53 pufferfish. How many fish are there in all? How might an estimate help you solve the problem? *Solve this problem any way you choose.*

0:01:44

When you are finished, turn to page 497, circle numbers 1-8, 9, 12, and 13, carefully rip it out and put it in your take-home folder.

If you finish early complete the Look Back.

Ask for Questions

SMART Notebook

File Edit View Insert Format Tools Add-ons Help

Thursday

A pet store has 162 goldfish 124 angelfish, and 53 pufferfish. What questions could you come up with?

00:03:39

5 minutes on your own
3 with a partner
All together

Outside Voice
Entire Class Can Hear You
Only People Near You Can Hear
Whisper
No Talking No Sound

Explore, Learn, Discover

Ask for Questions

pufferfish. What questions could you come up with?

How much more does the goldfish's has more than the angelfish?

What is the total of all fishes?

How much does the pufferfish and the goldfish have altogether?

How much does the angelfish and

Ask for Questions

What could the questions be now?

how much more is

blah blah blah than blah

blah blah?

Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Ask about ideas, not answers.

This can be really simple:

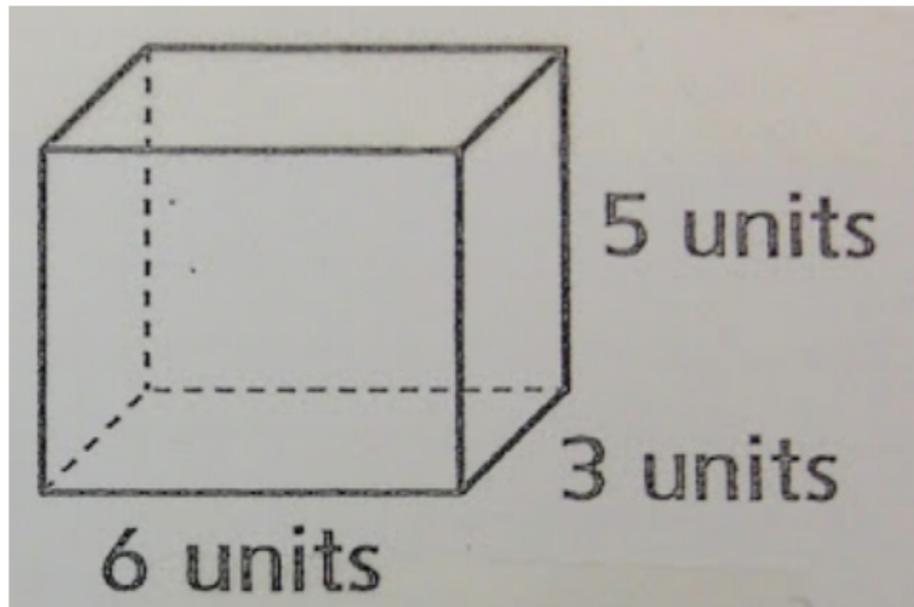
“Tell me something about number 7.”

instead of

“What's the answer to number 7?”

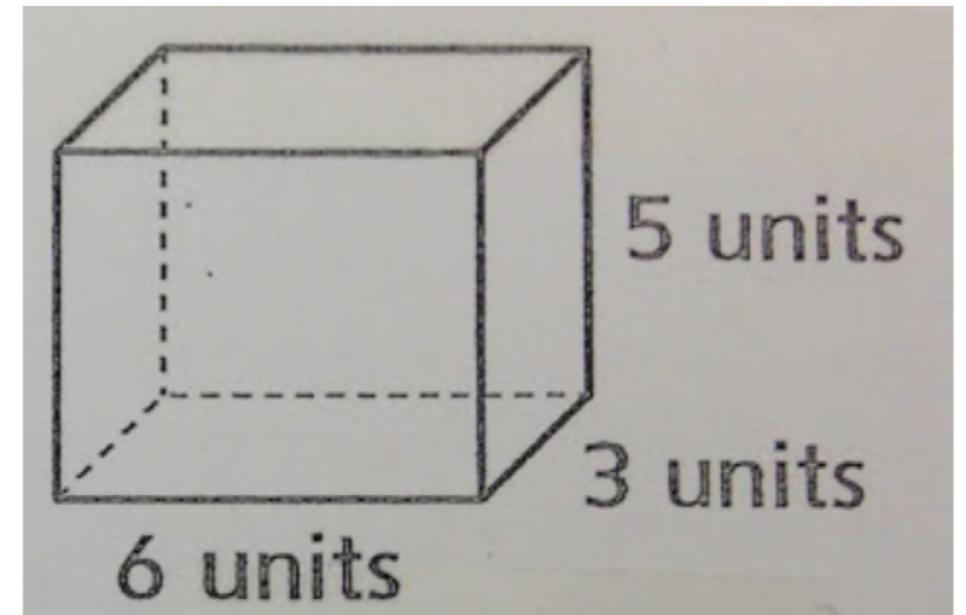
Ask About Ideas, Not Answers

It can be a little more complex:



Tell me everything you can about this figure.

instead of



Find the volume of the rectangular prism.

(from Joe Schwartz's blog, exit10a.blogspot.com, October 10, 2016)

Teacher Questions

“Why?”

“How do you know?”

“How did you decide?”

“Tell me more about that.”

“Phone in Pocket”

Are you asking *idea-focused* questions or *answer-focused* questions? Record yourself and find out!

#ToVForRatio

Your Most Important Job: Monitor for Sense-Making

Ways to Encourage Sense Making Rather Than Answer Getting

- Get rid of the question.
- Get rid of the question *and* the numbers.
- Give the answer.
- Ask about ideas, not answers.

Moment for Reflection and Personal (Possibly Public) Commitments

What's one thing you might try next week?

What's one thing you're wondering?

Thank you!

Annie Fetter

anniefetter@gmail.com, @MFAnnie

Slides and links to related resources will be available on my blog after the talk:

annie.mathematicalthinking.org